Rossby number

Rotating systems in the atmosphere can be described with the gradient wind equation. This equation holds the balance between the centrifugal force, the horizontal pressure gradient force (P) and the Coriolis force (C). As a start we repeat equation 7.16 from Wallace and Hobbs (page 283):

$$\vec{n}\frac{V^2}{R_T} = -\nabla\Phi - f\,\vec{k}\times\vec{V} \tag{9.1}$$

which can also be written in component form using natural coordinates:

$$\frac{V^2}{R_T} = -\frac{\partial\Phi}{\partial n} - fV \tag{9.2}$$

To simplify these equations we need to find the relative importance of individual terms so one can apply “scale analysis”. Relatively small terms can then be neglected. Note that the pressure gradient force can <u>never</u> be neglected as it is the only force that can drive horizontal motions in the atmosphere. The other terms of Equation 9.1 can be neglected under certain conditions.

In case the isohypses are straight and parallel (*i.e.* no centrifugal force) we are left with an equation that has been named *geostrophic balance*. In physical terms this would mean the radius of curvature is infinitely large ($R_T \rightarrow \pm\infty$), and thus the wind follows a straight line. Motions like this are described in Wallace and Hobbs equation 7.15 (page 281):

$$\overrightarrow{V_g} = \frac{1}{f}\left(\vec{k}\times\nabla\Phi\right) \tag{9.3}$$

This equation can be derived from Equation 9.1. It is easier if we put Equation 9.3 in component form and natural coordinates:

$$V_g = -\frac{1}{f}\frac{\partial\Phi}{\partial n} \tag{9.4}$$

To describe the flow around a ‘normal’ (anti)cyclone the full gradient wind equation is needed. The resulting wind speed is either subgeostrophic (for cyclones) or supergeostrophic (for anticyclones).

Neglecting the Coriolis force is permitted in cases with small horizontal scale. The remaining equation is a *cyclostrophic balance*. In this case the direction of the flow is not determined by the Coriolis force and can be both cyclonic and anticyclonic around a low pressure area (Figure 9.1). Using Equation 9.2 this would lead to

$$\frac{V^2}{R_T} = -\frac{\partial\Phi}{\partial n}$$

Equation 8.5 on page 352 of Wallace and Hobbs states the same result but for isobars instead of isohypses:

$$\frac{v^2}{r} = \frac{1}{\rho}\frac{\partial p}{\partial r} \tag{9.5}$$

Note that Wallace and Hobbs have replaced the $\vec{n}$ -coordinate with the $\vec{r}$ -coordinate which is in the opposite direction, hence the minus sign has disappeared. Also note that the geopotential has been rearranged to pressure (using equation 3.20 of Wallace and Hobbs, page 68).

To determine whether it is allowed to neglect the Coriolis force the *Rossby number* is often used. The Rossby number is a dimensionless [-] indicator of the relative importance of the Coriolis force to the centrifugal force. The formal definition is:

$$Ro \equiv \frac{F_{cf}}{C} = \frac{\frac{V^2}{R_T}}{f\,V} = \frac{V}{f\,R_T} \qquad (9.6)$$

Small values ($Ro \approx 0 - 0.1$) of the Rossby number indicate that the flow is nearly in geostrophic balance. Large values ($Ro > 100$) of the Rossby number indicate that cyclostrophic balance is valid.

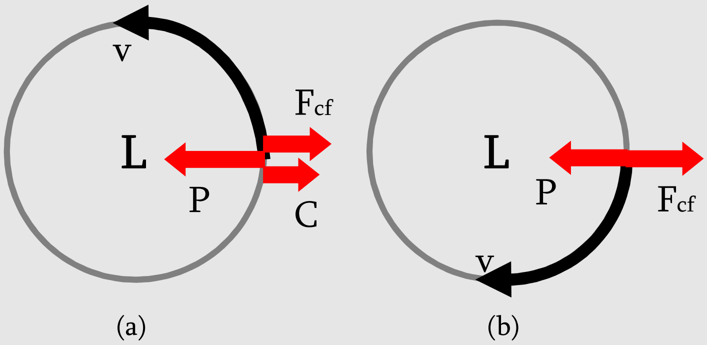

Figure 9.1 Force balance around an anti-clockwise turning Low pressure area (a) and a clockwise turning Low pressure area (b). Both in the Northern Hemisphere. The latter can only exist if the Coriolis force is small compared to the other forces (e.g. in dust devils).