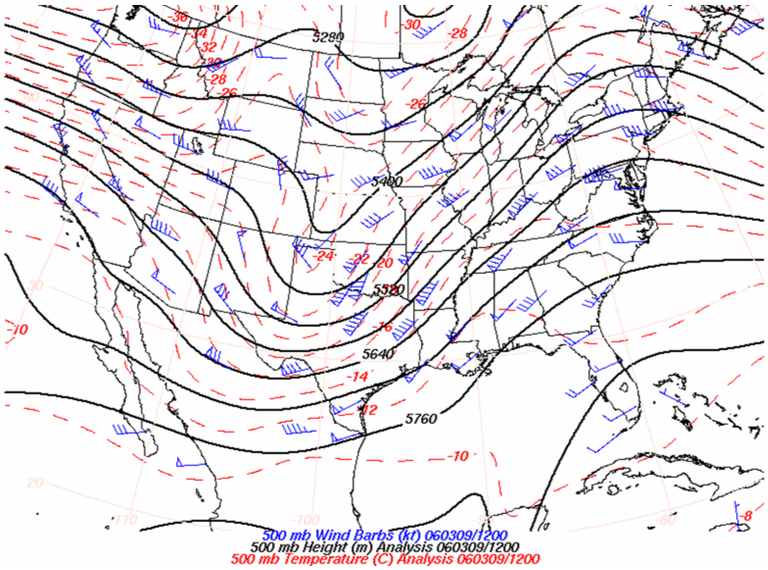
Quasi-Geostrophic (QG) Theory



Quasi-Geostrophic (QG) Theory

QG Theory

- Basic Idea
- Approximations and Validity
- QG Equations
- QG Reference

QG Prediction

- Basic Idea
- Estimating System Evolution
    - QG Height Tendency Equation
- Estimating Vertical Motion
    - QG Omega Equation

QG Theory: Basic Idea

Forecast Needs:

- The public desires information regarding temperature, humidity, precipitation, and wind speed and direction up to 7 days in advance across the entire country
- Such information is largely a function of the **evolving synoptic weather patterns** (i.e., surface pressure systems, fronts, and jet streams)

Four Forecast Methods:

Conceptual Models: **Based on numerous observations from past events**
Generalization of the synoptic patterns
Polar-Front theory, Norwegian Cyclone Model

Kinematic Approach: **Analyze current observations of wind, temperature, and moisture fields**
Assume clouds and precipitation occur when there is upward motion and an adequate supply of moisture
QG theory

Numerical models: **Based on integration of the primitive equations forward in time**
Require dense observations, and accurate physical parameterizations
User must compensate for erroneous initial conditions and model errors

Statistical models: **Use observations or numerical model output to infer the likelihood of of certain meteorological events**

QG Theory: Basic Idea

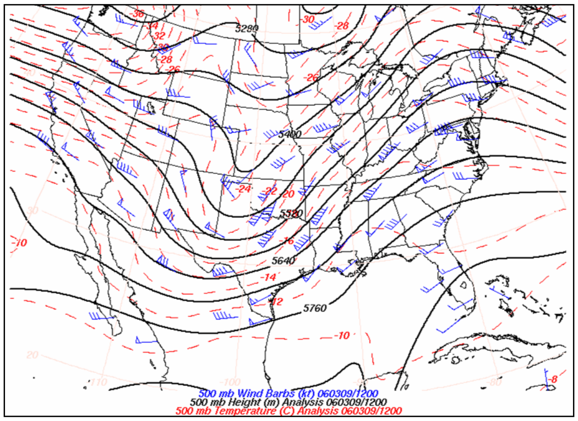
What will QG Theory do for us?

- It reveals how ***hydrostatic balance*** and ***geostrophic balance*** constrain and simplify atmospheric motions, but in a realistic manner
- It provides a simple framework within which we can understand and diagnose the ***evolution*** of three-dimensional synoptic-scale weather systems
- It helps us to understand how the mass fields (via horizontal temperature advection) and the momentum fields (via horizontal vorticity advection) interact to create vertical circulations that result in ***realistic synoptic-scale weather patterns***
- It offers physical insight into the ***forcing of vertical motion*** and the cloud/precipitation patterns associated with mid-latitude cyclones

QG Theory: Approximations and Validity

What do we already know?

- The primitive equations are quite complicated
- For mid-latitude synoptic-scale motions the horizontal winds are nearly geostrophic (i.e., they are **quasi-geostrophic**) above the surface
- We can use this fact to further simplify the equations, and still maintain accuracy



QG Theory: Approximations and Validity

- Start with:
  - Primitive equations in isobaric coordinates (to simplify the dynamics)
  - Hydrostatic Balance (valid for synoptic-scale flow)

$$\frac{Du}{Dt} = -g\frac{\partial z}{\partial x} + fv$$ **Zonal Momentum**

$$\frac{Dv}{Dt} = -g\frac{\partial z}{\partial y} - fu$$ **Meridional Momentum**

$$g\frac{\partial z}{\partial p} = -\frac{RT}{p}$$ **Hydrostatic Approximation**

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} = 0$$ **Mass Continuity**

$$\frac{DT}{Dt} = \omega\frac{RT}{pc_p} + \frac{1}{c_p}\frac{DQ}{Dt}$$ **Thermodynamic Energy**

$$p = \rho RT$$ **Equation of State**

QG Theory: Approximations and Validity

- Split the total horizontal velocity into geostrophic and ageostrophic components

$$u = u_g + u_a \qquad v = v_g + v_a$$

(u_g, v_g) → geostrophic → portion of the total wind in geostrophic balance
(u_a, v_a) → ageostrophic → portion of the total wind NOT in geostrophic balance

- Recall the horizontal equations of motion (isobaric coordinates):

$$\frac{Du}{Dt} = -g\frac{\partial z}{\partial x} + fv \qquad \frac{Dv}{Dt} = -g\frac{\partial z}{\partial y} - fu$$

where $\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + \omega\frac{\partial}{\partial p}$

QG Theory: Approximations and Validity

- Perform a scale analysis of the acceleration and Coriolis terms (construct ratios):

$$\frac{Dv/Dt}{fu} \qquad \frac{Du/Dt}{fv}$$

- For typical mid-latitude synoptic-scale systems:

$$\frac{Du/Dt}{fv} = \frac{(10\text{ms}^{-1})/10^5\text{s}}{(10^{-4}\text{s}^{-1})(10\text{ms}^{-1})} \sim 0.1$$

- Thus, we can assume:

$$\frac{Du}{Dt} << fv \quad \to\to \quad 0 \approx -g\frac{\partial z}{\partial x} + fv \quad \to\to \quad v \approx \frac{g}{f}\frac{\partial z}{\partial x}$$

$$\frac{Dv}{Dt} << fu \quad \to\to \quad 0 \approx -g\frac{\partial z}{\partial y} - fu \quad \to\to \quad u \approx -\frac{g}{f}\frac{\partial z}{\partial y}$$

- Since by definition (geostrophic balance)

$$v_g \equiv \frac{g}{f}\frac{\partial z}{\partial x} \qquad u_g \equiv -\frac{g}{f}\frac{\partial z}{\partial y}$$

We can also assume:

$$\to\to \quad u_g >> u_a \quad \to\to \quad u \approx u_g$$

$$v_g >> v_a \qquad v \approx v_g$$

QG Theory: Approximations and Validity

- If the ageostrophic component of the wind is not important then, we can assume:

$$\frac{Du}{Dt} \approx \frac{D_g u_g}{Dt} \qquad \frac{Dv}{Dt} \approx \frac{D_g v_g}{Dt} \qquad \text{where:} \quad \frac{D_g}{Dt} = \frac{\partial}{\partial t} + u_g \frac{\partial}{\partial x} + v_g \frac{\partial}{\partial y}$$

Note: The vertical advection term disappears from the total derivative

- This represents a significant simplification of the primitive equations and is the **primary simplification** in QG theory: Horizontal advection is accomplished by only the geostrophic winds

- What do our "new" equations of motion look like?

$$\frac{D_g u_g}{Dt} = -g\frac{\partial z}{\partial x} + fv \qquad \frac{D_g v_g}{Dt} = -g\frac{\partial z}{\partial y} - fu$$

- **What do we do with the total winds in the Coriolis accelerations?**

QG Theory: Approximations and Validity

- What do we do with the total winds in the Coriolis accelerations?
  - If we simply replace [u, v] with [u_g, v_g] then geostrophic balance is achieved on the right hand, and we have →→ $\frac{D_g u_g}{Dt} = 0$ $\frac{D_g v_g}{Dt} = 0$
  - We do **NOT** want this → **Some accelerations are required so the flow can evolve**

- Therefore, the total wind is **retained** in the Coriolis acceleration:

$$\frac{D_g u_g}{Dt} = -g\frac{\partial z}{\partial x} + f(v_g + v_a)$$

$$\frac{D_g v_g}{Dt} = -g\frac{\partial z}{\partial y} - f(u_g + u_a)$$

Accelerations in the geostrophic wind result entirely from ageostrophic flow associated with the Coriolis force

QG Theory: Approximations and Validity

- We can, however, make an assumption about the Coriolis parameter (f) that will ultimately simplify our full system of equations:
  - Approximate the Coriolis parameter with a Taylor Series expansion:

$$f = f_0 + \frac{\partial f}{\partial y} y \quad \rightarrow\rightarrow \quad f = f_0 + \beta y$$

where: f_o is the Coriolis parameter at a constant reference latitude

$\beta = \frac{\partial f}{\partial y}$ is the constant meridional gradient in the Coriolis parameter

- Perform a scale analysis on the two terms, we find: $f_0 >> \beta y$

- We can thus re-write our geostrophic balance equations as:

$$v_g \equiv \frac{g}{f}\frac{\partial z}{\partial x} \quad \rightarrow\rightarrow \quad v_g \equiv \frac{g}{f_0}\frac{\partial z}{\partial x}$$

$$u_g \equiv -\frac{g}{f}\frac{\partial z}{\partial y} \quad \rightarrow\rightarrow \quad u_g \equiv -\frac{g}{f_0}\frac{\partial z}{\partial y}$$

QG Equations: Equations of Motion

- Let's recap:

$$u = u_g + u_a \qquad v = v_g + v_a \qquad f = f_0 + \beta y \qquad v_g \equiv \frac{g}{f_0}\frac{\partial z}{\partial x} \qquad u_g \equiv -\frac{g}{f_0}\frac{\partial z}{\partial y}$$

$$\frac{\partial u_g}{\partial t} + u_g\frac{\partial u_g}{\partial x} + v_g\frac{\partial u_g}{\partial y} = -g\frac{\partial z}{\partial x} + (f_0 + \beta y)(v_g + v_a)$$

$$\frac{\partial v_g}{\partial t} + u_g\frac{\partial v_g}{\partial x} + v_g\frac{\partial v_g}{\partial y} = -g\frac{\partial z}{\partial y} - (f_0 + \beta y)(u_g + u_a)$$

- QG Theory neglects the following from the primitive equations of motion:
  - Friction
  - Horizontal advection of momentum by the ageostrophic wind
  - Vertical advection of momentum
  - Local changes in the ageostrophic wind
  - Advection of the ageostrophic momentum by the geostrophic wind

QG Equations: Continuity Equation

- Start with primitive form of the mass continuity equation in isobaric coordinates:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} = 0$$

- Substitute in: $v = v_g + v_a$, $u = u_g + u_a$ and then using: $v_g \equiv \frac{g}{f_0}\frac{\partial z}{\partial x}$, $u_g \equiv -\frac{g}{f_0}\frac{\partial z}{\partial y}$

- One can easily show that the **geostrophic flow is nondivergent**, or

$$\frac{\partial u_g}{\partial x} + \frac{\partial v_g}{\partial y} = 0$$

- Thus, the QG continuity is:

$$\frac{\partial u_a}{\partial x} + \frac{\partial v_a}{\partial y} + \frac{\partial \omega}{\partial p} = 0$$

The vertical velocity (ω) only depends on the ageostrophic component of the flow

QG Equations: Thermodynamic Equation

- Start with primitive form of the thermodynamic equation in isobaric coordinates:

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + \omega\frac{\partial T}{\partial p} = \omega\frac{RT}{pc_p} + \frac{1}{c_p}\frac{DQ}{Dt}$$

- We can combine the two terms containing vertical motion (ω) such that,

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \omega\sigma\frac{p}{R} + \frac{1}{c_p}\frac{DQ}{Dt}$$ where: $$\sigma = -\frac{RT}{p\theta}\frac{\partial \theta}{\partial p}$$

- Next, we apply the primary QG approximation ($u \approx u_g$ and $v \approx v_g$),

$$\frac{\partial T}{\partial t} + u_g\frac{\partial T}{\partial x} + v_g\frac{\partial T}{\partial y} = \omega\sigma\frac{p}{R} + \frac{1}{c_p}\frac{DQ}{Dt}$$

- Finally, we neglect the diabatic heating (Q) term [for now…we will return to this later]

$$\frac{\partial T}{\partial t} + u_g\frac{\partial T}{\partial x} + v_g\frac{\partial T}{\partial y} = \omega\sigma\frac{p}{R}$$

Circulation and Vorticity

Circulation: The tendency for a group of air parcels to rotate
If an **area** of atmosphere is of interest, you compute the circulation

Vorticity: The tendency for the wind shear at a given point to induce rotation
If a **point** in the atmosphere is of interest, you compute the vorticity

Planetary Vorticity: Vorticity associated with the Earth's rotation

$$f \equiv 2\Omega \sin\phi$$

Relative Vorticity: Vorticity associated with 3D shear in the wind field

$$\nabla \times \mathbf{V} \equiv \mathbf{i}\left(\frac{\partial \omega}{\partial y} - \frac{\partial v}{\partial p}\right) + \mathbf{j}\left(\frac{\partial u}{\partial p} - \frac{\partial \omega}{\partial x}\right) + \mathbf{k}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

Only the vertical component of vorticity (the **k** component) is of interest for large-scale (synoptic) meteorology

$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

Absolute Vorticity: The sum of relative and planetary vorticity

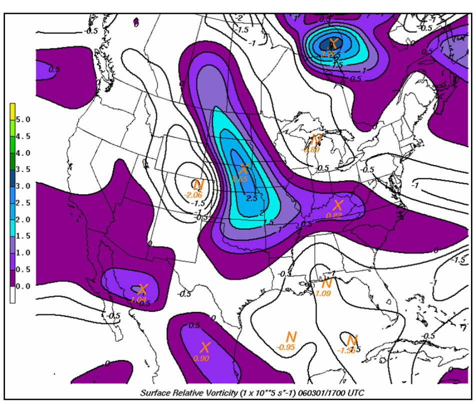
$$\eta \equiv \zeta + f$$

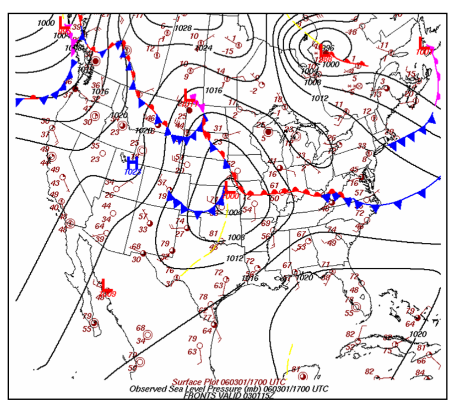
Circulation and Vorticity

Vorticity:

Positive Vorticity: Associated with cyclonic (counterclockwise) circulations in the Northern Hemisphere

Negative Vorticity: Associated with anticyclonic (clockwise) circulations in the Northern Hemisphere



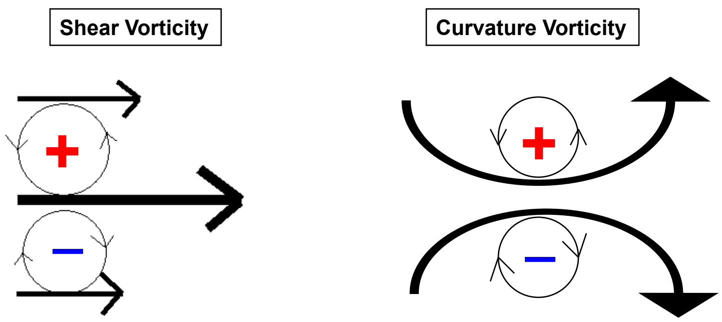


Circulation and Vorticity

Vorticity:

Shear Vorticity: Associated with gradients along local straight-line wind maxima

Curvature Vorticity: Associated with the turning of flow along a stream line



Vorticity Equation

Describes the factors that alter the magnitude of the absolute vorticity with time

Derivation: Start with the horizontal momentum equations (in isobaric coordinates)

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \omega\frac{\partial u}{\partial p} = -g\frac{\partial z}{\partial x} + fv \quad \textbf{Zonal Momentum}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \omega\frac{\partial v}{\partial p} = -g\frac{\partial z}{\partial y} - fu \quad \textbf{Meridional Momentum}$$

Take $\frac{\partial}{\partial x}$ of the meridional equation and subtract $\frac{\partial}{\partial y}$ of the zonal equation

After use of the product rule, some simplifications, and cancellations:

$$\frac{\partial \zeta}{\partial t} + u\frac{\partial \zeta}{\partial x} + v\frac{\partial \zeta}{\partial y} + \omega\frac{\partial \zeta}{\partial p} + v\frac{\partial f}{\partial y} = -(\zeta + f)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + \left(\frac{\partial \omega}{\partial y}\frac{\partial u}{\partial p} - \frac{\partial \omega}{\partial x}\frac{\partial v}{\partial p}\right)$$

Vorticity Equation

What do the terms represent?

$$\frac{\partial \zeta}{\partial t} = -u\frac{\partial \zeta}{\partial x} - v\frac{\partial \zeta}{\partial y} - \omega\frac{\partial \zeta}{\partial p} - v\frac{\partial f}{\partial y} - (\zeta + f)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + \left(\frac{\partial \omega}{\partial y}\frac{\partial u}{\partial p} - \frac{\partial \omega}{\partial x}\frac{\partial v}{\partial p}\right)$$

Black	Local rate of change of relative vorticity	$\sim 10^{-10}$
Grey	Horizontal advection of relative vorticity	$\sim 10^{-10}$
Magenta	Vertical advection of relative vorticity	$\sim 10^{-11}$
Green	Meridional advection of planetary vorticity	$\sim 10^{-10}$
Red	Divergence Term	$\sim 10^{-9}$
Cyan	Tilting Terms	$\sim 10^{-11}$

What are the significant terms? → Scale analysis and neglect of "small" terms yields:

$$\frac{\partial \zeta}{\partial t} = -u\frac{\partial \zeta}{\partial x} - v\frac{\partial \zeta}{\partial y} - v\frac{\partial f}{\partial y} - (\zeta + f)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$$

Vorticity Equation

Physical Explanation of Significant Terms:

$$\frac{\partial \zeta}{\partial t} = -u\frac{\partial \zeta}{\partial x} - v\frac{\partial \zeta}{\partial y} - v\frac{\partial f}{\partial y} - (\zeta + f)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$$

Horizontal Advection of Relative Vorticity

- The local relative vorticity will increase (decrease) if positive (negative) relative vorticity is advected toward the location → Positive Vorticity Advection (PVA) and
  → Negative Vorticity Advection (NVA)
- PVA often leads to a decrease in surface pressure (intensification of surface lows)

Meridional Advection of Planetary Vorticity

- The local relative vorticity will decrease (increase) if the local flow is southerly (northerly) due to the advection of planetary vorticity (minimum at Equator; maximum at poles)

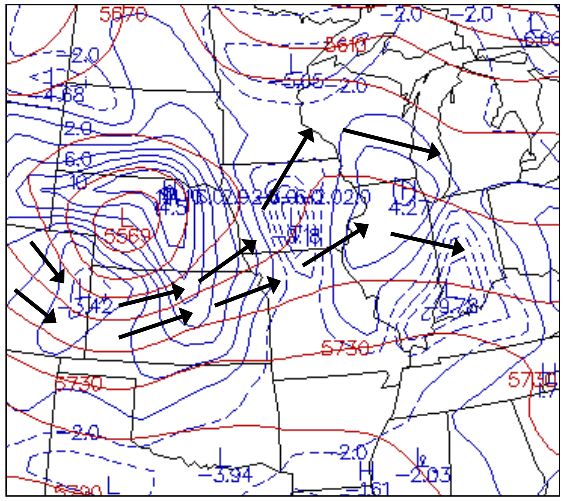
Divergence Term

- The local relative vorticity will increase (decrease) if local convergence (divergence) exists

QG Equations: Vorticity Equation

- More on **Term 2** (Relative Vorticity Advection):

Remember that the geostrophic flow (**implied** by black arrows) is parallel to the geopotential height contours (in red)



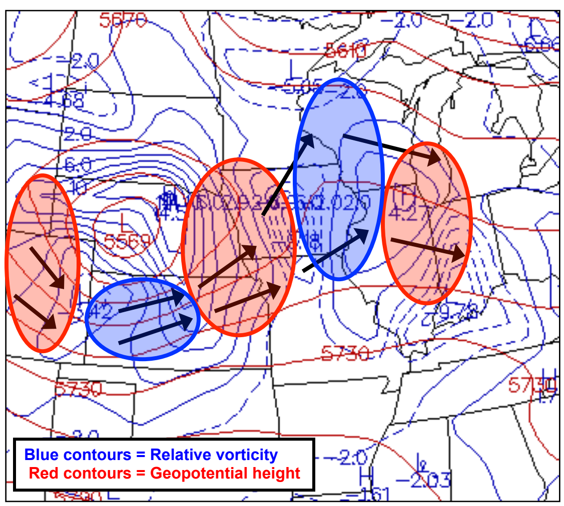
QG Equations: Vorticity Equation

- More on **Term 2** (Relative Vorticity Advection):

Remember that the geostrophic flow (**implied** by black arrows) is parallel to the geopotential height contours (in red)

Note the regions of positive vorticity advection (**PVA**), or regions of **increasing local vorticity**

Note the regions of negative vorticity advection (**NVA**), or regions of **decreasing local vorticity**



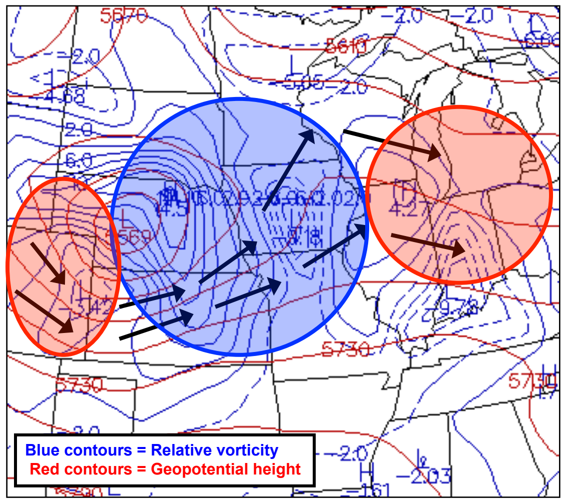
QG Equations: Vorticity Equation

- More on **Term 3** (Planetary Vorticity Advection):

Remember that the geostrophic flow (**implied** by black arrows) is parallel to the geopotential height contours (in red)

Note the regions of **southward** flow that correspond to regions of **increasing local vorticity**

Note the regions of **northward** flow that correspond to regions of **decreasing local vorticity**



QG Equations: Vorticity Equation

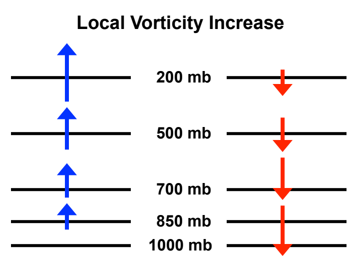
- More on **Term 4** (Vertical Stretching of Air Columns):

Compare vertical motions at multiple pressure levels

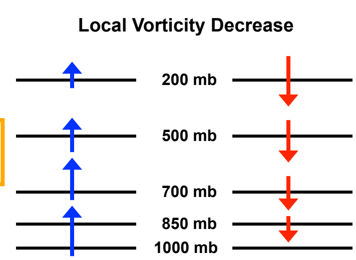
Increasing vertical motions with height (ω becoming more negative) correspond to locations of increasing local vorticity

Decreasing vertical motions with height (ω becoming more positive) correspond to locations of decreasing local vorticity

$$f_0 \frac{\partial \omega}{\partial p} > 0$$



$$f_0 \frac{\partial \omega}{\partial p} < 0$$



QG Equations: Reference

Summary of the QG equations:

$u = u_g + u_a$ $v = v_g + v_a$ Decomposition of Total Winds

$f = f_0 + \beta y$ Coriolis Approximation

$u_g \equiv -\frac{g}{f_0}\frac{\partial z}{\partial y}$ $v_g \equiv \frac{g}{f_0}\frac{\partial z}{\partial x}$ Geostrophic Balance

$$\frac{\partial u_g}{\partial t} + u_g\frac{\partial u_g}{\partial x} + v_g\frac{\partial u_g}{\partial y} = -g\frac{\partial z}{\partial x} + (f_0 + \beta y)(v_g + v_a)$$ Zonal Momentum Equation

$$\frac{\partial v_g}{\partial t} + u_g\frac{\partial v_g}{\partial x} + v_g\frac{\partial v_g}{\partial y} = -g\frac{\partial z}{\partial y} - (f_0 + \beta y)(u_g + u_a)$$ Meridional Momentum Equation

$$\frac{\partial u_a}{\partial x} + \frac{\partial v_a}{\partial y} + \frac{\partial \omega}{\partial p} = 0$$ Continuity Equation

$$\frac{\partial u_g}{\partial x} + \frac{\partial v_g}{\partial y} = 0$$ Geostrophic Nondivergence

$$\frac{\partial T}{\partial t} + u_g\frac{\partial T}{\partial x} + v_g\frac{\partial T}{\partial y} = \omega\sigma\frac{p}{R}$$ Thermodynamic Equation

$$\sigma = -\frac{RT}{p\theta}\frac{\partial \theta}{\partial p}$$ Static Stability

$$\frac{\partial \zeta_g}{\partial t} + u_g\frac{\partial \zeta_g}{\partial x} + v_g\frac{\partial \zeta_g}{\partial y} = f_0\frac{\partial \omega}{\partial p} - \beta v_g$$ Vorticity Equation

$$\zeta_g = \frac{\partial v_g}{\partial x} - \frac{\partial u_g}{\partial y}$$ Vorticity

QG Theory: Summary

QG Theory: Limiting Assumptions

- Geostrophic Balance
- Hydrostatic Balance
- Horizontal advection by the geostrophic winds only
- No variation in the Coriolis parameter

- No Friction
- No orographic effects
- No diabatic heating / cooling
- No spatial or temporal changes in static stability

Note: We will discuss how to compensate for these latter four limitations as we progress through each topic

QG and Forecasting

Most meteorological forecasts:

- Focus on Temperature, Winds, and Precipitation (amount and type)
- Are largely a function of the **evolving synoptic-scale weather patterns**

Quasi-Geostrophic Theory:

- Makes further simplifying assumptions about the large-scale dynamics
- Diagnostic methods to estimate: Changes in large-scale surface pressure
  Changes in large-scale temperature (thickness)
  Regions of large-scale vertical motion
- Despite the simplicity, it provides accurate estimates of large-scale changes
- Will provide the basic analysis framework for remainder of the semester

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QG Prediction: Basic Idea

Forecast Needs:

- The public desires information regarding temperature, humidity, precipitation, and wind speed and direction up to 7 days in advance across the entire country
- Such information is largely a function of the **evolving synoptic weather patterns** (i.e., surface pressure systems, fronts, and jet streams)

Forecast Method:

Kinematic Approach: **Analyze current observations of wind, temperature, and moisture fields**
Assume clouds and precipitation occur when there is upward motion and an adequate supply of moisture
QG theory

QG Prediction:

- System Evolution: Related to changes in the mass and momentum fields
  ** Diagnose changes in the local geopotential height field from the observed distributions of absolute vorticity advection and temperature advection
- Vertical Motion: Related to all forces that induce vertical motion
  ** Estimate synoptic-scale vertical motions from the observed distributions of absolute vorticity and temperature advection

QG Prediction: System Evolution

The Geopotential-Height Tendency (χ):

- In order to predict system evolution we wish to examine *changes* in the local height field
- Therefore, we wish to develop a single *prognostic* equation for geopotential height

- First, we define a local change (or tendency) in geopotential-height:

$$\chi = \frac{\partial \Phi}{\partial t} \quad \text{where} \quad \Phi = gz$$

- Next we need to define our two primary *prognostic* variables – **vorticity** (ζ_g) and **temperature** (T) – in terms of geopotential height…

$$\frac{\partial \zeta_g}{\partial t} + u_g \frac{\partial \zeta_g}{\partial x} + v_g \frac{\partial \zeta_g}{\partial y} = f_0 \frac{\partial \omega}{\partial p} - \beta v_g$$

Vorticity Equation

$$\frac{\partial T}{\partial t} + u_g \frac{\partial T}{\partial x} + v_g \frac{\partial T}{\partial y} = \omega \sigma \frac{p}{R}$$

Thermodynamic Equation

QG Prediction: System Evolution

Expressing Vorticity in terms of Geopotential Height:

• Begin with the definition of geostrophic relative vorticity:

$$\zeta_g = \frac{\partial v_g}{\partial x} - \frac{\partial u_g}{\partial y} \qquad u_g \equiv -\frac{g}{f_0}\frac{\partial z}{\partial y} \qquad v_g \equiv \frac{g}{f_0}\frac{\partial z}{\partial x}$$

• Substitute using the geostrophic wind relations, and one can easily show:

$$\zeta_g = \frac{1}{f_0}\nabla_p^2\Phi$$

• Remember that the Laplacian is just the second derivative:

$$\nabla_p^2\Phi = \nabla_p \bullet \nabla_p\Phi = \frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2}$$

QG Prediction: System Evolution

Expressing Temperature in terms of Geopotential Height:

• Begin with the hydrostatic relation in isobaric coordinates:

$$g\frac{\partial z}{\partial p} = -\frac{RT}{p}$$

• Using the definition of geopotential and some algebra, one can easily show:

$$T = -\frac{p}{R}\frac{\partial \Phi}{\partial p}$$

QG Prediction: System Evolution

The Geopotential-Height Tendency (χ):

• We can now define local changes in vorticity and temperature in terms of the local height tendency (on constant pressure surfaces)

$$\frac{\partial \zeta_g}{\partial t} = \frac{\partial}{\partial t}\left(\frac{1}{f_0}\nabla_p^2 \Phi\right) = \frac{1}{f_0}\nabla_p^2 \chi$$

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial t}\left(-\frac{p}{R}\frac{\partial \Phi}{\partial p}\right) = -\frac{p}{R}\frac{\partial \chi}{\partial p}$$

Note: These two relationships are very "powerful" and will be used to physically interpret terms in the height tendency and omega equations

Note: The latter relation is equivalent to the hypsometric equation

• We can now substitute the new relationships into the QG vorticity and thermodynamic equations:

$$\frac{1}{f_0}\nabla_p^2 \chi + u_g \frac{\partial \zeta_g}{\partial x} + v_g \frac{\partial \zeta_g}{\partial y} = f_0 \frac{\partial \omega}{\partial p} - \beta v_g$$

$$-\frac{p}{R}\frac{\partial \chi}{\partial p} + u_g \frac{\partial T}{\partial x} + v_g \frac{\partial T}{\partial y} = \omega\sigma \frac{p}{R}$$

Note: These two equations will be used to obtain both the height tendency and the omega equations

QG Prediction: System Evolution

The Quasigeostrophic Height-Tendency Equation:

- We can now derive a ***single*** prognostic equation for Φ (diagnostic equation for χ) by combining our new vorticity and thermodynamic equations:

$$\frac{1}{f_0}\nabla_p^2 \chi + u_g \frac{\partial \zeta_g}{\partial x} + v_g \frac{\partial \zeta_g}{\partial y} = f_0 \frac{\partial \omega}{\partial p} - \beta v_g$$

$$-\frac{p}{R}\frac{\partial \chi}{\partial p} + u_g \frac{\partial T}{\partial x} + v_g \frac{\partial T}{\partial y} = \omega\sigma \frac{p}{R}$$

- To do this, we need to eliminate the vertical motion (ω) from both equations

Step 1: Apply the operator $-\frac{f_0^2}{\sigma}\frac{\partial}{\partial p}\left(\frac{R}{p}\right)$ to the thermodynamic equation

Step 2: Multiply the vorticity equation by f_0

Step 3: Add the results of Steps 1 and 2

After a lot of math, we get the resulting diagnostic equation……

QG Prediction: System Evolution

The BASIC Quasigeostrophic Height-Tendency Equation:

$$\underbrace{\left(\nabla_p^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\chi}_{\text{Term A}} = \underbrace{f_0\left[-\mathbf{v}_g \bullet \nabla_p\left(\zeta_g + f\right)\right]}_{\text{Term B}} + \underbrace{-\frac{f_0^2}{\sigma}\frac{\partial}{\partial p}\left(\frac{R}{p}\left(-\mathbf{v}_g \bullet \nabla_p T\right)\right)}_{\text{Term C}}$$

- To obtain an ***actual value*** for χ (the ideal goal), we would need to compute the forcing terms (Terms B and C) from the three-dimensional wind and temperature fields, and then invert the operator in Term A using appropriate boundary conditions
- This is not a simple task (*forecasters never do this*)…..

- Rather, we can ***infer the sign and relative magnitude*** of χ through simple inspection of the three-dimensional absolute vorticity and temperature fields (*forecasters do this all the time…*)
- Thus, let's examine the physical interpretation of each term….

Note: This is ***not*** the same form of the QG height-tendency equation shown in Holton
While equivalent to "Holton's version", this version is easier to physically understand and is the version used in the Bluestein text

QG Prediction: System Evolution

The BASIC Quasigeostrophic Height-Tendency Equation:

$$\underbrace{\left(\nabla_p^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\chi}_{\text{Term A}} = \underbrace{f_0\left[-\mathbf{v}_g \bullet \nabla_p\left(\zeta_g + f\right)\right]}_{\text{Term B}} + \underbrace{-\frac{f_0^2}{\sigma}\frac{\partial}{\partial p}\left(\frac{R}{p}\left(-\mathbf{v}_g \bullet \nabla_p T\right)\right)}_{\text{Term C}}$$

Term A: Local Tendency in Geopotential Height

- For synoptic-scale atmospheric waves, this term is **proportional to –χ**
- However, if we incorporate the negative sign into our physical interpretation, which we will do, we can just think of this term as the local height tendency
- Thus, this term is **our goal** – a qualitative estimate of the local height change

QG Prediction: System Evolution

The BASIC Quasigeostrophic Height-Tendency Equation:

$$\underbrace{\left(\nabla_p^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\chi}_{\text{Term A}} = \underbrace{f_0\left[-\mathbf{v_g} \bullet \nabla_p\left(\zeta_g + f\right)\right]}_{\text{Term B}} + \underbrace{-\frac{f_0^2}{\sigma}\frac{\partial}{\partial p}\left(\frac{R}{p}\left(-\mathbf{v_g} \bullet \nabla_p T\right)\right)}_{\text{Term C}}$$

Term B: Horizontal Advection of Absolute Vorticity

- Positive vorticity advection (**PVA**) causes local vorticity increases:

PVA $u_g\frac{\partial \zeta_g}{\partial x} + v_g\frac{\partial \zeta_g}{\partial y} > 0 \rightarrow \frac{\partial \zeta_g}{\partial t} > 0$

- From our relationship between ζ_g and χ, we know that PVA is equivalent to:

$\frac{\partial \zeta_g}{\partial t} = \frac{1}{f_0}\nabla_p^2\chi$ therefore: **PVA** $\rightarrow \nabla_p^2\chi > 0$ or: **PVA** $\rightarrow \chi < 0$

- From our interpretation of Term A, we thus know that **PVA** leads to **height falls**

QG Prediction: System Evolution

The BASIC Quasigeostrophic Height-Tendency Equation:

$$\underbrace{\left(\nabla_p^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\chi}_{\text{Term A}} = \underbrace{f_0\left[-\mathbf{v_g} \bullet \nabla_p\left(\zeta_g + f\right)\right]}_{\text{Term B}} + \underbrace{-\frac{f_0^2}{\sigma}\frac{\partial}{\partial p}\left(\frac{R}{p}\left(-\mathbf{v_g} \bullet \nabla_p T\right)\right)}_{\text{Term C}}$$

Term B: Horizontal Advection of Absolute Vorticity

- Negative vorticity advection (**NVA**) causes local vorticity decreases:

NVA $u_g\frac{\partial \zeta_g}{\partial x} + v_g\frac{\partial \zeta_g}{\partial y} < 0 \rightarrow \frac{\partial \zeta_g}{\partial t} < 0$

- From our relationship between ζ_g and χ, we know that NVA is equivalent to:

$\frac{\partial \zeta_g}{\partial t} = \frac{1}{f_0}\nabla_p^2\chi$ therefore: **NVA** $\rightarrow \nabla_p^2\chi < 0$ or: **NVA** $\rightarrow \chi > 0$

- From our interpretation of Term A, we thus know that **NVA** leads to **height rises**

QG Prediction: System Evolution

The BASIC Quasigeostrophic Height-Tendency Equation:

$$\underbrace{\left(\nabla_p^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\chi}_{\text{Term A}} = \underbrace{f_0\left[-\mathbf{v}_g \bullet \nabla_p\left(\zeta_g + f\right)\right]}_{\text{Term B}} + \underbrace{-\frac{f_0^2}{\sigma}\frac{\partial}{\partial p}\left(\frac{R}{p}\left(-\mathbf{v}_g \bullet \nabla_p T\right)\right)}_{\text{Term C}}$$

Term B: Horizontal Advection of Absolute Vorticity

- Often the primary forcing in the upper troposphere (500 mb and above)

- Term is equal to zero at local maxima and minima of absolute vorticity
- **If the maxima and minima are collocated with trough and ridge axes**, (which is often the case) this term **can not change the strength** of the system by increasing or decreasing the amplitude of the trough or ridge
- Thus, this term is **primarily responsible for system motion**

- However, this is not always the case (as you will see in homework exercises…)

QG Prediction: System Evolution

IMPORTANT POINT:

Troughs and ridges do <u>not</u> move in DIRECT response to advection

Rather, troughs and ridges move in response to local geopotential height changes caused by vorticity (and temperature) advection.

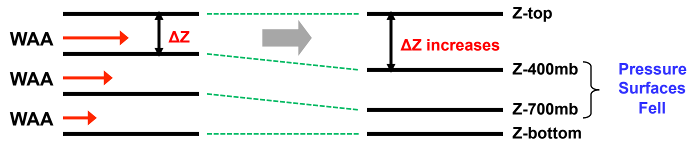
QG Prediction: System Evolution

The BASIC Quasi-geostrophic Height-Tendency Equation:

$$\underbrace{\left(\nabla_p^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\chi}_{\text{Term A}} = \underbrace{f_0\left[-\mathbf{v}_g \bullet \nabla_p\left(\zeta_g + f\right)\right]}_{\text{Term B}} + \underbrace{-\frac{f_0^2}{\sigma}\frac{\partial}{\partial p}\left(\frac{R}{p}\left(-\mathbf{v}_g \bullet \nabla_p T\right)\right)}_{\text{Term C}}$$

Term C: Change in Temperature Advection with "Height"

- Consider a three layer atmosphere where the temperature advection increases with height, or warm air advection (**WAA**) is strongest in the upper layer



- The greater temperature increase aloft will produce the greatest thickness increase in the upper layer and lower the pressure surfaces (or heights) in the lower levels
- Therefore an **increase in WAA advection with height** leads to **height falls**

QG Prediction: System Evolution

The BASIC Quasi-geostrophic Height-Tendency Equation:

$$\underbrace{\left(\nabla_p^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\chi}_{\text{Term A}} = \underbrace{f_0\left[-\mathbf{v}_g \bullet \nabla_p\left(\zeta_g + f\right)\right]}_{\text{Term B}} + \underbrace{-\frac{f_0^2}{\sigma}\frac{\partial}{\partial p}\left(\frac{R}{p}\left(-\mathbf{v}_g \bullet \nabla_p T\right)\right)}_{\text{Term C}}$$

Term C: Change in Temperature Advection with "Height"

- Possible **height fall** scenarios:

  WAA in upper level
  CAA in lower levels

  No temperature advection in upper levels
  CAA in lower levels

  Strong **WAA** in upper levels
  Weak **WAA** in lower levels

  Weak **CAA** in upper levels
  Strong **CAA** in lower levels

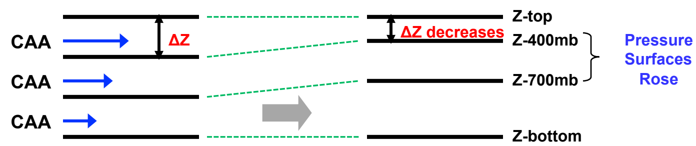
QG Prediction: System Evolution

The BASIC Quasi-geostrophic Height-Tendency Equation:

$$\underbrace{\left(\nabla_p^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\chi}_{\text{Term A}} = \underbrace{f_0\left[-\mathbf{v}_g \bullet \nabla_p\left(\zeta_g + f\right)\right]}_{\text{Term B}} + \underbrace{-\frac{f_0^2}{\sigma}\frac{\partial}{\partial p}\left(\frac{R}{p}\left(-\mathbf{v}_g \bullet \nabla_p T\right)\right)}_{\text{Term C}}$$

Term C: Change in Temperature Advection with "Height"

- Again, consider a three layer atmosphere where the temperature advection decreases with height, or cold air advection (**CAA**) is strongest in the upper layer



- The greater temperature decrease aloft will produce the greatest thickness decrease in the upper layer and raise the pressure surfaces (or heights) in the lower levels
- Therefore an **increase in CAA advection with height** leads to **height rises**

QG Prediction: System Evolution

The BASIC Quasi-geostrophic Height-Tendency Equation:

$$\underbrace{\left(\nabla_p^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\chi}_{\text{Term A}} = \underbrace{f_0\left[-\mathbf{v}_g \bullet \nabla_p\left(\zeta_g + f\right)\right]}_{\text{Term B}} + \underbrace{-\frac{f_0^2}{\sigma}\frac{\partial}{\partial p}\left(\frac{R}{p}\left(-\mathbf{v}_g \bullet \nabla_p T\right)\right)}_{\text{Term C}}$$

Term C: Change in Temperature Advection with "Height"

- Possible **height rise** scenarios:
  - **CAA** in upper level
    WAA in lower levels
  - No temperature advection in upper levels
    WAA in lower levels
  - Strong **CAA** in upper levels
    Weak **CAA** in lower levels
  - Weak **WAA** in upper levels
    Strong **WAA** in lower levels

QG Prediction: System Evolution

The BASIC Quasi-geostrophic Height-Tendency Equation:

$$\underbrace{\left(\nabla_p^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\chi}_{\text{Term A}} = \underbrace{f_0\left[-\mathbf{v_g}\bullet\nabla_p\left(\zeta_g + f\right)\right]}_{\text{Term B}} + \underbrace{-\frac{f_0^2}{\sigma}\frac{\partial}{\partial p}\left(\frac{R}{p}\left(-\mathbf{v_g}\bullet\nabla_p T\right)\right)}_{\text{Term C}}$$

Term C: Change in Temperature Advection with "Height"

- Often the primary forcing in the lower atmosphere (below 500 mb)

- Term is equal to zero at local maxima and minima of temperature
- Since the temperature maxima and minima are often located ***between*** the trough and ridge axes, significant temperature advection (or height changes) can occur ***at*** the axes and thus **can change the system strength or intensity**

Note: In the absence of diabatic heating (i.e. clouds), synoptic-scale systems must have non-zero temperature advection in order to intensify

QG Prediction: System Evolution

The BASIC Quasi-geostrophic Height-Tendency Equation:

$$\underbrace{\left(\nabla_p^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\chi}_{\text{Term A}} = \underbrace{f_0\left[-\mathbf{v_g}\bullet\nabla_p\left(\zeta_g + f\right)\right]}_{\text{Term B}} + \underbrace{-\frac{f_0^2}{\sigma}\frac{\partial}{\partial p}\left(\frac{R}{p}\left(-\mathbf{v_g}\bullet\nabla_p T\right)\right)}_{\text{Term C}}$$

Summary and Application Tips:

- You must consider the effects of both **Term B** and **Term C** using multiple levels
- If the vorticity maxima/minima are not collocated with trough/ridge axes, then this term will contribute to system intensity change as well as motion
- If the vorticity advection patterns change with height, then expect the the system "tilt" to change with time (become more "tilted" or more "stacked")
- If large (small) changes in temperature advection with height are observed, then you should expect large (small) system intensity changes
- Combine the results from both terms to estimate the three-dimensional motion and intensity changes of a system

QG Prediction: System Evolution

Summary and Final Comments:

- The QG height-tendency equation is a **prognostic** equation (forecasting):
  - Can be used to ***predict*** the future pattern of pressure surface heights
- Use the QG height-tendency equation in a prognostic setting:
  - Diagnose the **synoptic–scale** contribution to the height field evolution
  - Predict the formation, movement, and evolution of synoptic waves and systems
- Use of the QG height-tendency equation in a **diagnostic** setting (research):
  - Diagnose the synoptic-scale contribution to height field changes
  - Compare to the total (or observed) height field changes
  - The difference can be used to infer the mesoscale and convective-scale contributions to the evolution of synoptic systems

QG Prediction: Vertical Motion

Estimating vertical motion in the atmosphere:

Our Challenge:

- We do not observe vertical motion
- Intimately linked to clouds and precipitation
- Actual vertical motions are often several orders of magnitude smaller than their collocated horizontal air motions [w ~ 0.01 → 10 m/s] [u,v ~ 10 → 100 m/s]
- Synoptic-scale vertical motions must be estimated from widely-spaced observations (i.e. the rawindsonde network) every 12-hours

Methods:

• Kinematic Method	Integrate the Continuity Equation Very sensitive to small errors in winds measurements
• Adiabatic Method	From the thermodynamic equation Very sensitive to temperature tendencies (difficult to observe) Difficult to incorporate impacts of diabatic heating
▪ QG Omega Equation	Least sensitive to small observational errors Widely believed to be the best method

QG Prediction: Vertical Motion

The Quasi-geostrophic Omega Equation:

- We can also derive a ***single*** diagnostic equation for ω by, again, combining our vorticity and thermodynamic equations (the height-tendency versions from before):

$$\frac{1}{f_0}\nabla_p^2\chi + u_g\frac{\partial\zeta_g}{\partial x} + v_g\frac{\partial\zeta_g}{\partial y} = f_0\frac{\partial\omega}{\partial p} - \beta v_g$$

$$-\frac{p}{R}\frac{\partial\chi}{\partial p} + u_g\frac{\partial T}{\partial x} + v_g\frac{\partial T}{\partial y} = \omega\sigma\frac{p}{R}$$

- To do this, we need to eliminate the height tendency (χ) from both equations

Step 1: Apply the operator $-\frac{f_0}{\sigma}\frac{\partial}{\partial p}$ to the vorticity equation

Step 2: Apply the operator $\frac{R}{p\sigma}\nabla_p^2$ to the thermodynamic equation

Step 3: Subtract the result of Step 1 from the result of Step 2

After a lot of math, we get the resulting diagnostic equation……

QG Prediction: Vertical Motion

The BASIC Quasi-geostrophic Omega Equation:

$$\underbrace{\left(\nabla_p^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega}_{\text{Term A}} = \underbrace{-\frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[-\mathbf{v_g}\bullet\nabla_p\left(\zeta_g + f\right)\right]}_{\text{Term B}} + \underbrace{-\frac{R}{\sigma p}\nabla_p^2\left(-\mathbf{v_g}\bullet\nabla_p T\right)}_{\text{Term C}}$$

- To obtain an ***actual value*** for **ω** (the ideal goal), we would need to compute the forcing terms (Terms B and C) from the three-dimensional wind and temperature fields, and then invert the operator in Term A using appropriate boundary conditions
- Again, this is not a simple task (*forecasters never do this*)…..

- Rather, we can ***infer the sign and relative magnitude*** of **ω** through simple inspection of the three-dimensional absolute vorticity and temperature fields (*forecasters do this all the time…*)
- Thus, let's examine the physical interpretation of each term….

Note: Again, this is ***not*** the same form of the QG omega equation shown in Holton
While equivalent to "Holton's version", this version is easier to physically understand and is the version used in the Bluestein text

QG Prediction: Vertical Motion

The BASIC Quasi-geostrophic Omega Equation:

$$\left(\nabla_p^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega = -\frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[-\mathbf{v}_g \bullet \nabla_p\left(\zeta_g + f\right)\right] + -\frac{R}{\sigma p}\nabla_p^2\left(-\mathbf{v}_g \bullet \nabla_p T\right)$$

Term A — Term B — Term C

Term A: Local Vertical Motion

- For synoptic-scale atmospheric waves, this term is **proportional to –ω**
- Again, if we incorporate the negative sign into our physical interpretation, which we will do, we can just think of this term as the vertical motion
- Thus, this term is **our goal** – a qualitative estimate of the deep –layer synoptic-scale vertical motion at a particular location

QG Prediction: Vertical Motion

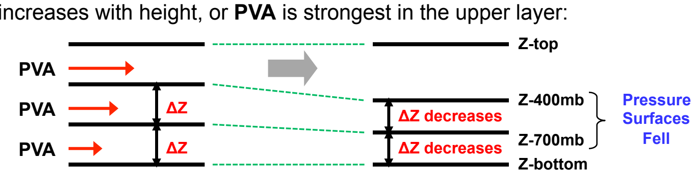
The BASIC Quasi-geostrophic Omega Equation:

$$\left(\nabla_p^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega = -\frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[-\mathbf{v}_g \bullet \nabla_p\left(\zeta_g + f\right)\right] + -\frac{R}{\sigma p}\nabla_p^2\left(-\mathbf{v}_g \bullet \nabla_p T\right)$$

Term A — Term B — Term C

Term B: Change in Absolute Vorticity Advection with "Height"

- Recall, positive vorticity advection (**PVA**) leads to local height falls
- Consider a three-layer atmosphere where cyclonic vorticity advection increases with height, or **PVA** is strongest in the upper layer:



- Hydrostatic balance (and the hypsometric equation) requires ALL changes in thickness (ΔZ) to be accompanied by temperature changes…

QG Prediction: Vertical Motion

The BASIC Quasi-geostrophic Omega Equation:

$$\underbrace{\left(\nabla_p^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega}_{\text{Term A}} = \underbrace{-\frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[-\mathbf{v}_g \bullet \nabla_p\left(\zeta_g + f\right)\right]}_{\text{Term B}} + \underbrace{-\frac{R}{\sigma p}\nabla_p^2\left(-\mathbf{v}_g \bullet \nabla_p T\right)}_{\text{Term C}}$$

Term B: Change in Absolute Vorticity Advection with "Height"

- These thickness decreases (height falls) were ***not*** a result of temperature changes
- Thus, in order to maintain hydrostatic balance, the thickness decreases must be accompanied by a temperature decrease
- In the absence of temperature advection and diabatic cooling, only adiabatic cooling associated with rising motion can create this required temperature decrease
- Therefore, an **increase in PVA with height** will induce **rising motion**

QG Prediction: Vertical Motion

The BASIC Quasi-geostrophic Omega Equation:

$$\underbrace{\left(\nabla_p^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega}_{\text{Term A}} = \underbrace{-\frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[-\mathbf{v}_g \bullet \nabla_p\left(\zeta_g + f\right)\right]}_{\text{Term B}} + \underbrace{-\frac{R}{\sigma p}\nabla_p^2\left(-\mathbf{v}_g \bullet \nabla_p T\right)}_{\text{Term C}}$$

Term B: Change in Absolute Vorticity Advection with "Height"

- Possible **rising motion** scenarios:

  PVA in upper levels
  NVA in lower levels

  PVA in upper levels
  No vorticity advection in lower levels

  Strong **PVA** in upper levels
  Weak **PVA** in lower levels

  Weak **NVA** in upper levels
  Strong **NVA** in lower levels

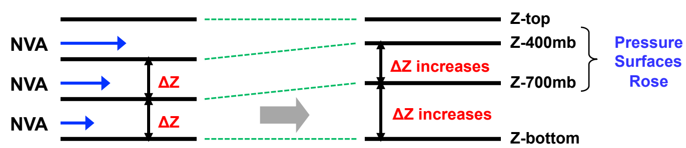
QG Prediction: Vertical Motion

The BASIC Quasi-geostrophic Omega Equation:

$$\underbrace{\left(\nabla_p^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega}_{\text{Term A}} = \underbrace{-\frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[-\mathbf{v}_g \bullet \nabla_p\left(\zeta_g + f\right)\right]}_{\text{Term B}} + \underbrace{-\frac{R}{\sigma p}\nabla_p^2\left(-\mathbf{v}_g \bullet \nabla_p T\right)}_{\text{Term C}}$$

Term B: Change in Absolute Vorticity Advection with "Height"

- Recall, negative vorticity advection (**NVA**) leads to local height rises
- Consider a three-layer atmosphere where anticyclonic vorticity advection increases with height, or **NVA** is strongest in the upper layer:



- Hydrostatic balance (and the hypsometric equation) requires ALL changes in thickness (ΔZ) to be accompanied by temperature changes…

QG Prediction: Vertical Motion

The BASIC Quasi-geostrophic Omega Equation:

$$\underbrace{\left(\nabla_p^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega}_{\text{Term A}} = \underbrace{-\frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[-\mathbf{v}_g \bullet \nabla_p\left(\zeta_g + f\right)\right]}_{\text{Term B}} + \underbrace{-\frac{R}{\sigma p}\nabla_p^2\left(-\mathbf{v}_g \bullet \nabla_p T\right)}_{\text{Term C}}$$

Term B: Change in Absolute Vorticity Advection with "Height"

- These thickness increases (height rises) were ***not*** a result of temperature changes
- Thus, in order to maintain hydrostatic balance, the thickness increases must be accompanied by a temperature increase

- In the absence of temperature advection and diabatic heating, only adiabatic heating associated with sinking motion can create this required temperature increase

- Therefore, an **increase in NVA with height** will induce **sinking motion**

QG Prediction: Vertical Motion

The BASIC Quasi-geostrophic Omega Equation:

$$\underbrace{\left(\nabla_p^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega}_{\text{Term A}} = \underbrace{-\frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[-\mathbf{v}_g \bullet \nabla_p\left(\zeta_g + f\right)\right]}_{\text{Term B}} + \underbrace{-\frac{R}{\sigma p}\nabla_p^2\left(-\mathbf{v}_g \bullet \nabla_p T\right)}_{\text{Term C}}$$

Term B: Change in Absolute Vorticity Advection with "Height"

- Possible **sinking motion** scenarios:

  NVA in upper levels
  PVA in lower levels

  NVA in upper levels
  No vorticity advection in lower levels

  Strong **NVA** in upper levels
  Weak **NVA** in lower levels

  Weak **PVA** in upper levels
  Strong **PVA** in lower levels

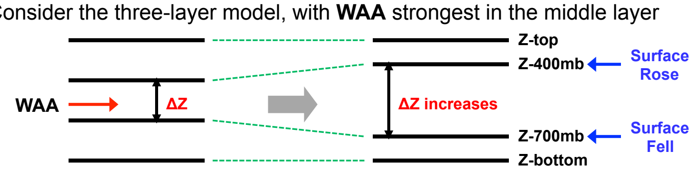
QG Prediction: Vertical Motion

The BASIC Quasi-geostrophic Omega Equation:

$$\underbrace{\left(\nabla_p^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega}_{\text{Term A}} = \underbrace{-\frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[-\mathbf{v}_g \bullet \nabla_p\left(\zeta_g + f\right)\right]}_{\text{Term B}} + \underbrace{-\frac{R}{\sigma p}\nabla_p^2\left(-\mathbf{v}_g \bullet \nabla_p T\right)}_{\text{Term C}}$$

Term C: Horizontal Temperature Advection

- Warm air advection (**WAA**) leads to local temperature increases
- Consider the three-layer model, with **WAA** strongest in the middle layer



- Under the constraint of geostrophic balance, local height rises (falls) require a change in the local pressure gradient, a change in the local geostrophic wind, and thus a local decrease (increase) in geostrophic vorticity…..

QG Prediction: Vertical Motion

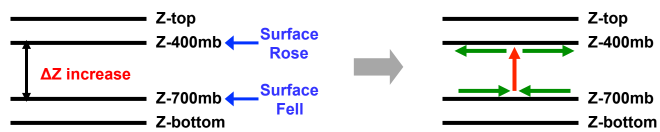
The BASIC Quasi-geostrophic Omega Equation:

$$\underbrace{\left(\nabla_p^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega}_{\text{Term A}} = \underbrace{-\frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[-\mathbf{v}_g \bullet \nabla_p\left(\zeta_g + f\right)\right]}_{\text{Term B}} + \underbrace{-\frac{R}{\sigma p}\nabla_p^2\left(-\mathbf{v}_g \bullet \nabla_p T\right)}_{\text{Term C}}$$

Term C: Horizontal Temperature Advection

- These height changes where ***not*** a result of changes in geostrophic vorticity
- Thus, in order to maintain geostrophic balance in the absence of vorticity advection, local height rises must be accompanied by divergence (which decreases vorticity) and height falls must be accompanied by convergence (which increases vorticity)
- Mass continuity then requires rising motion through layer

- Therefore, **WAA** will induce **rising motion**



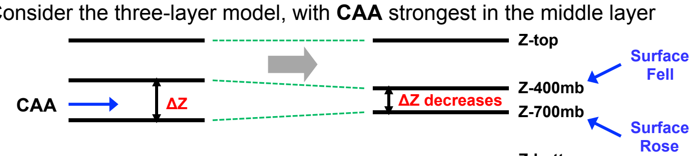
QG Prediction: Vertical Motion

The BASIC Quasi-geostrophic Omega Equation:

$$\underbrace{\left(\nabla_p^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega}_{\text{Term A}} = \underbrace{-\frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[-\mathbf{v}_g \bullet \nabla_p\left(\zeta_g + f\right)\right]}_{\text{Term B}} + \underbrace{-\frac{R}{\sigma p}\nabla_p^2\left(-\mathbf{v}_g \bullet \nabla_p T\right)}_{\text{Term C}}$$

Term C: Horizontal Temperature Advection

- Cold air advection (**CAA**) leads to local temperature decreases
- Consider the three-layer model, with **CAA** strongest in the middle layer



- Under the constraint of geostrophic balance, local height rises (falls) require a change in the local pressure gradient, a change in the local geostrophic wind, and thus a local decrease (increase) in geostrophic vorticity…..

QG Prediction: Vertical Motion

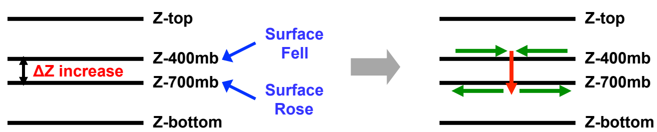
The BASIC Quasi-geostrophic Omega Equation:

$$\underbrace{\left(\nabla_p^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega}_{\text{Term A}} = \underbrace{-\frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[-\mathbf{v}_g \cdot \nabla_p\left(\zeta_g + f\right)\right]}_{\text{Term B}} + \underbrace{-\frac{R}{\sigma p}\nabla_p^2\left(-\mathbf{v}_g \cdot \nabla_p T\right)}_{\text{Term C}}$$

Term C: Horizontal Temperature Advection

- These height changes where ***not*** a result of changes of geostrophic vorticity
- Thus, in order to maintain geostrophic balance in the absence of vorticity advection, local height rises must be accompanied by divergence (which decreases vorticity) and height falls must be accompanied by convergence (which increases vorticity)
- Mass continuity then requires sinking motion through layer

- Therefore, **CAA** will induce **sinking motion**



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Summary and Application Tips:

- You must consider the effects of both **Term B** and **Term C** at multiple levels
- If large (small) changes in the vorticity advection with height are observed, then you should expect large (small) vertical motions
- The stronger the temperature advection, the stronger the vertical motion
- If WAA (CAA) is observed at several consecutive pressure levels, expect a deep layer of rising (sinking) motion
- Opposing expectations in vertical motion from the two terms at a given location will alter the total vertical motion pattern

QG Prediction: Vertical Motion

Summary and Final Comments:

- The QG omega equation is a **diagnostic** equation:
  - The equation does ***not predict*** future vertical motion patterns
  - The forcing functions (Terms B and C) do not ***cause*** the expected responses, with an implied time lag between the forcing and the response
  - The responses are *instantaneous*
  - The responses are a direct result of the atmosphere maintaining hydrostatic and geostrophic balance at the time of the forcing
- Use of the QG omega equation in a **diagnostic** setting (forecasting):
  - Diagnose the **synoptic–scale** vertical motion pattern, and assume rising motion corresponds to clouds and precipitation when ample moisture is available
  - Compare to the observed patterns → Infer mesoscale contributions
- Use of the QG omega equation in a **limited prognostic** setting (forecasting):
  - Diagnose the **synoptic-scale contribution** to the total vertical motion, cloud, and precipitation patterns predicted at a future time by a numerical model
  - Help distinguish between regions of persistent precipitation (synoptic scale) and more sporadic precipitation (mesoscale)

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