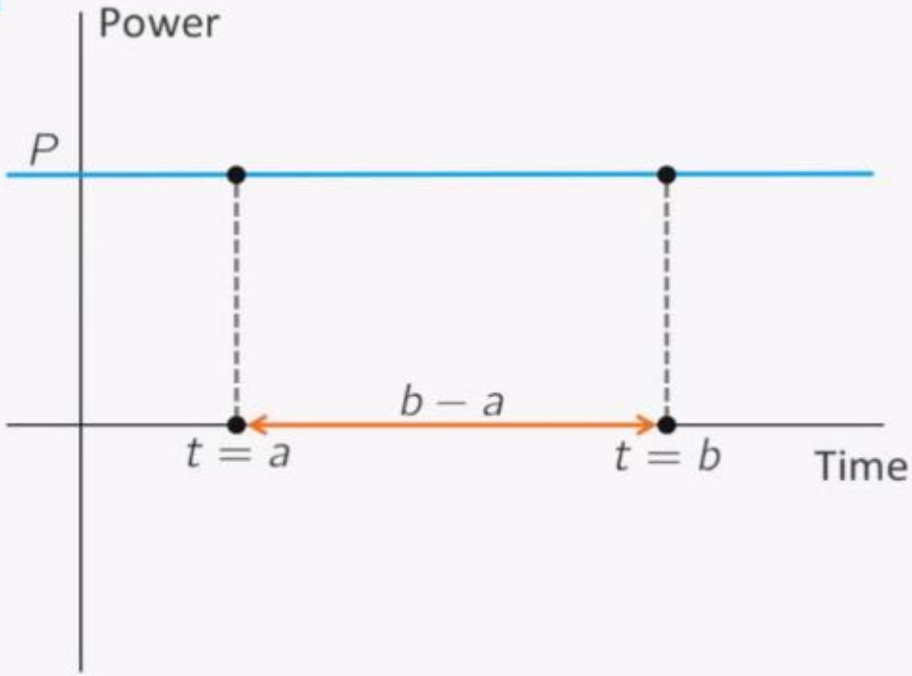
Integrais

DelftX: CalcSP01x Pre-University Calculus (Self-Paced)

Week 6. Integration

6.2 Concept of integration

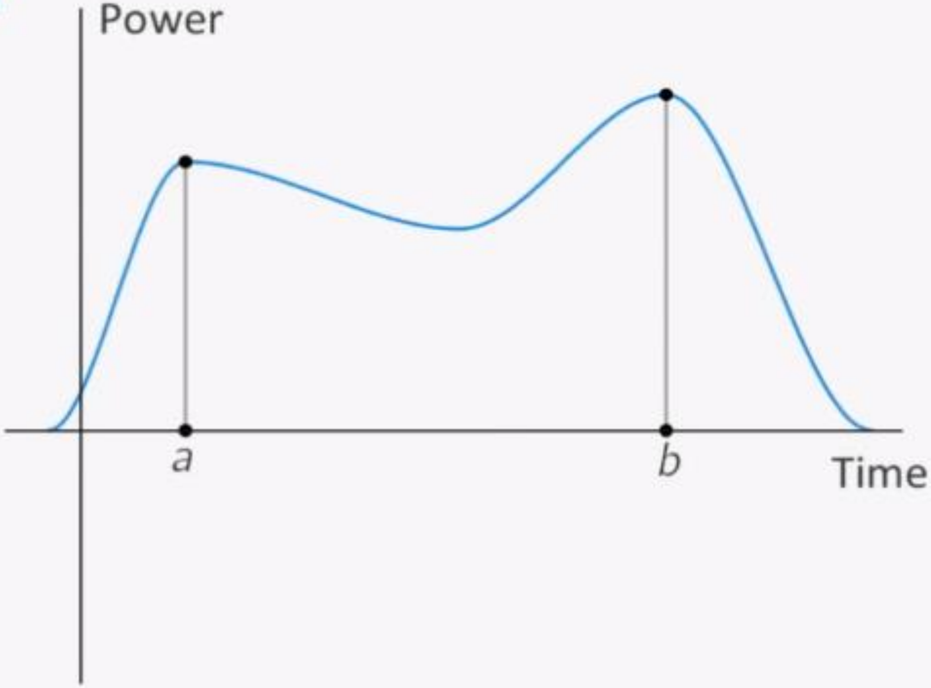
Constant power



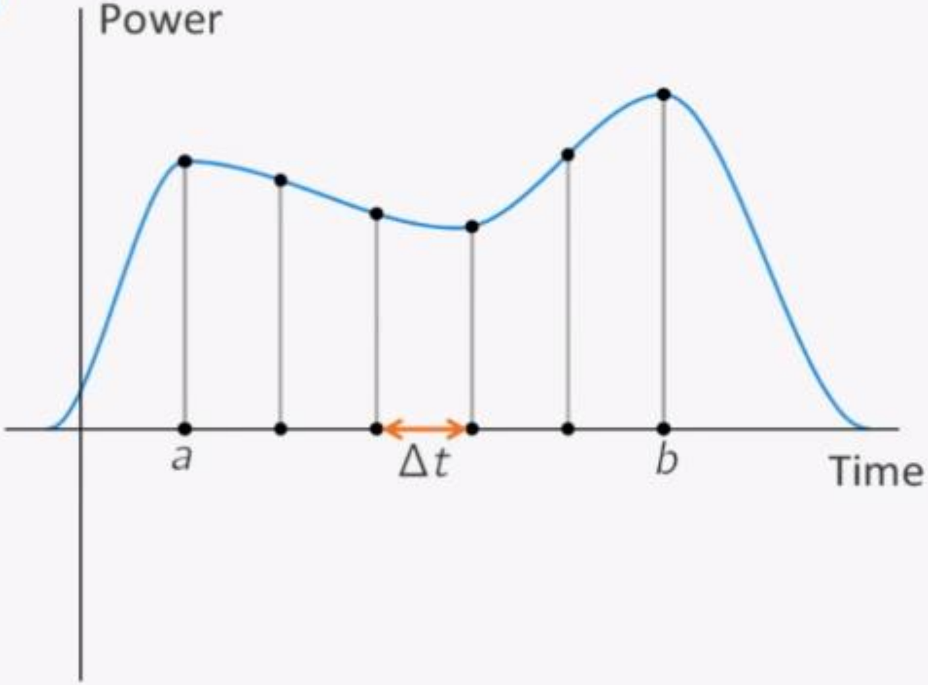
Energy consumption

$$= P \times (b - a)$$

Non-constant power

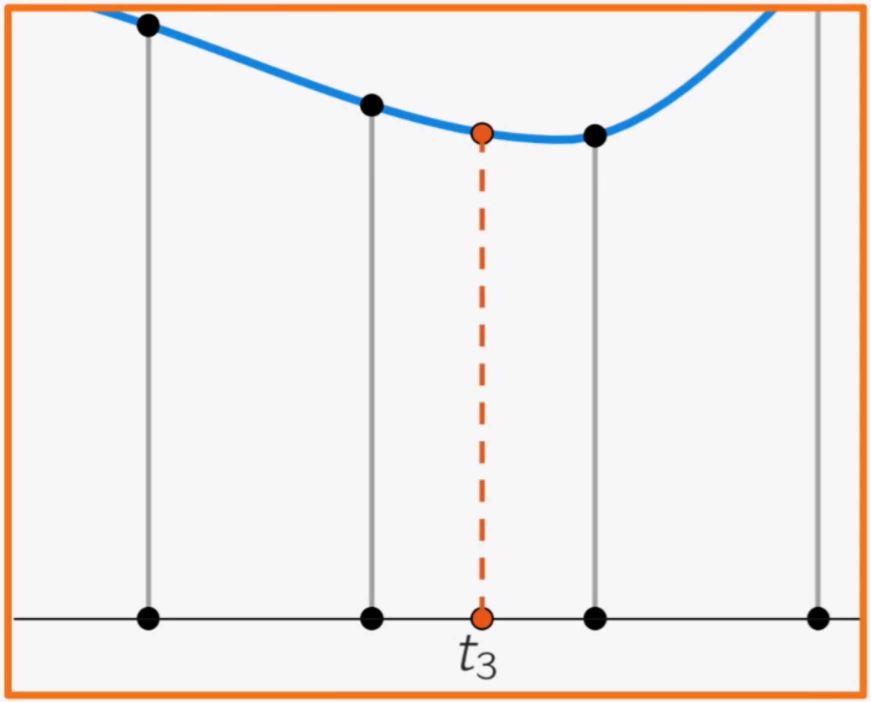


Non-constant power



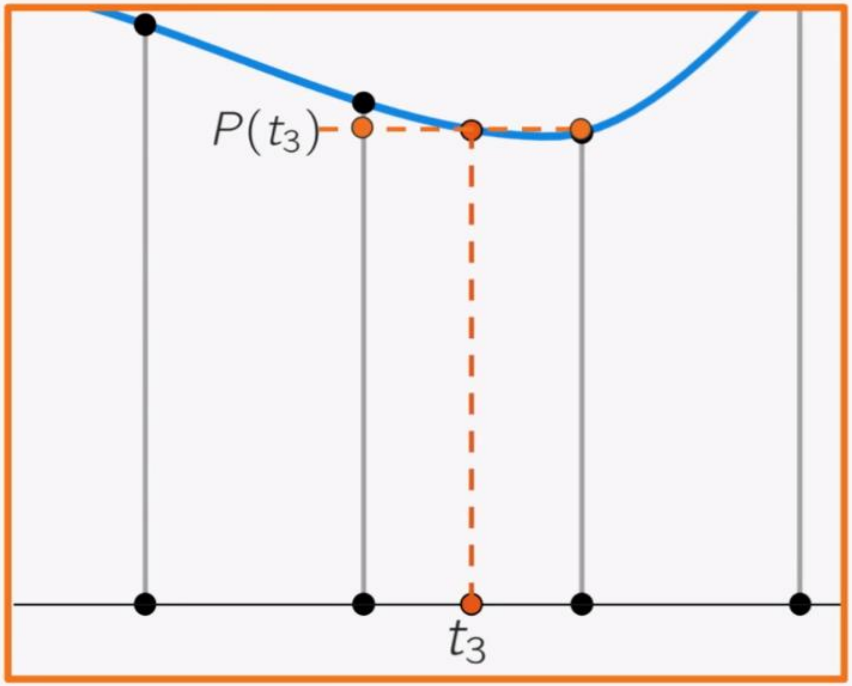
- Subintervals of size Δt

Non-constant power



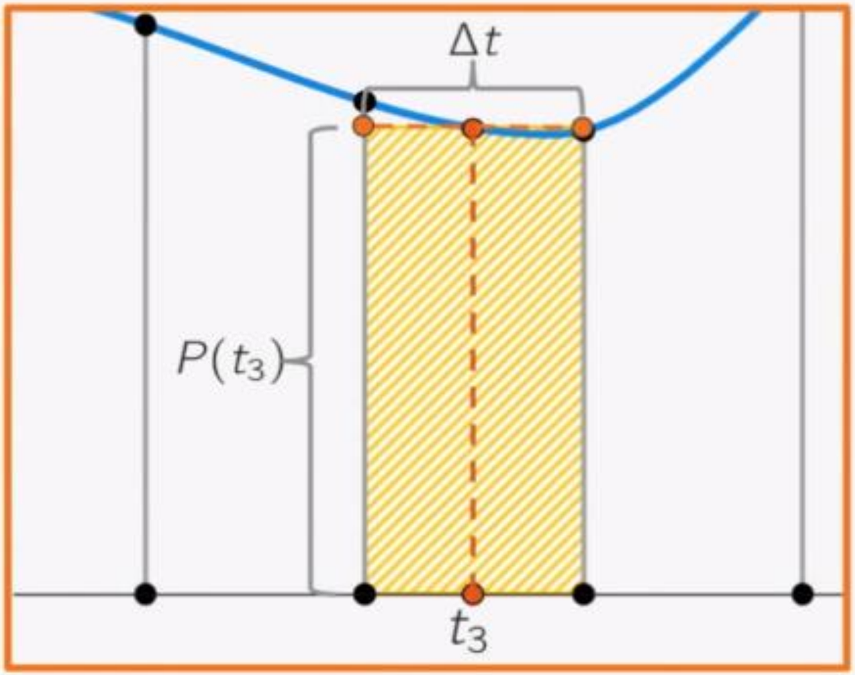
- Subintervals of size Δt
- Point in subinterval

Non-constant power



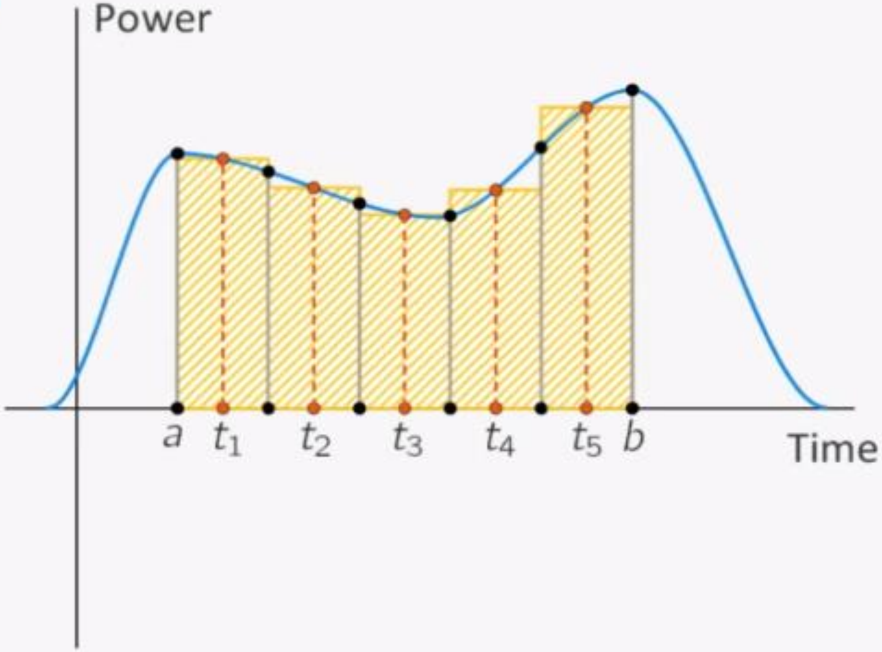
- Subintervals of size Δt
- Point in subinterval
- Energy consumption in subinterval $\approx P(t_3) \cdot \Delta t$

Non-constant power



- Subintervals of size Δt
- Point in subinterval
- Energy consumption in subinterval $\approx P(t_3) \cdot \Delta t$

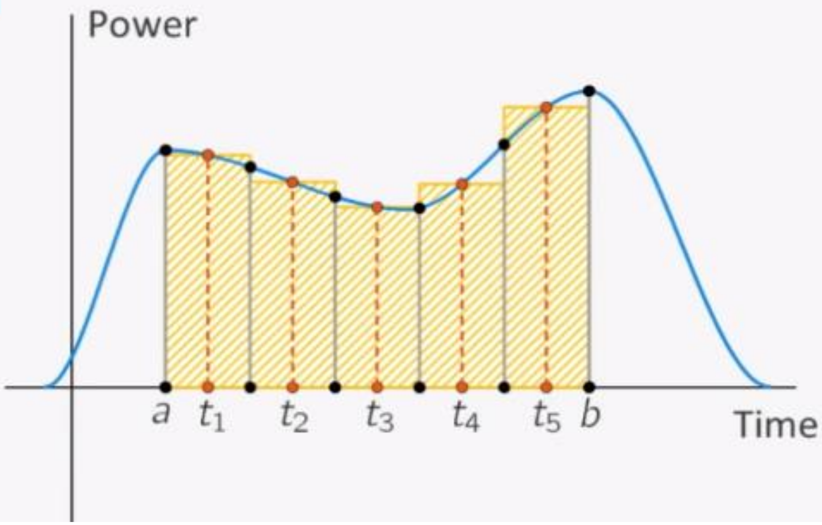
Non-constant power



- Subintervals of size Δt
- Point in *each* subinterval
- Total energy

$\approx P(t_1)\cdot\Delta t + P(t_2)\cdot\Delta t + \ldots$

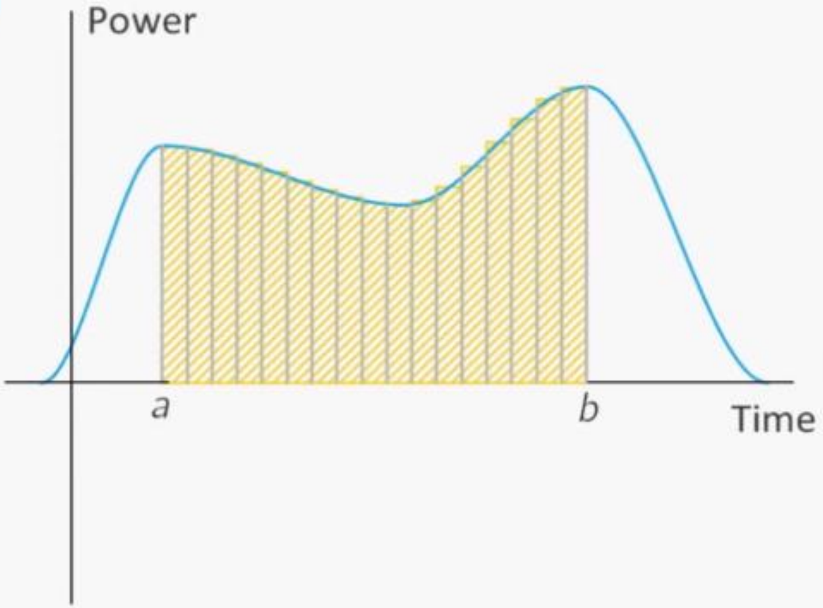
Non-constant power



- Subintervals of size Δt
- Point in *each* subinterval
- Total energy

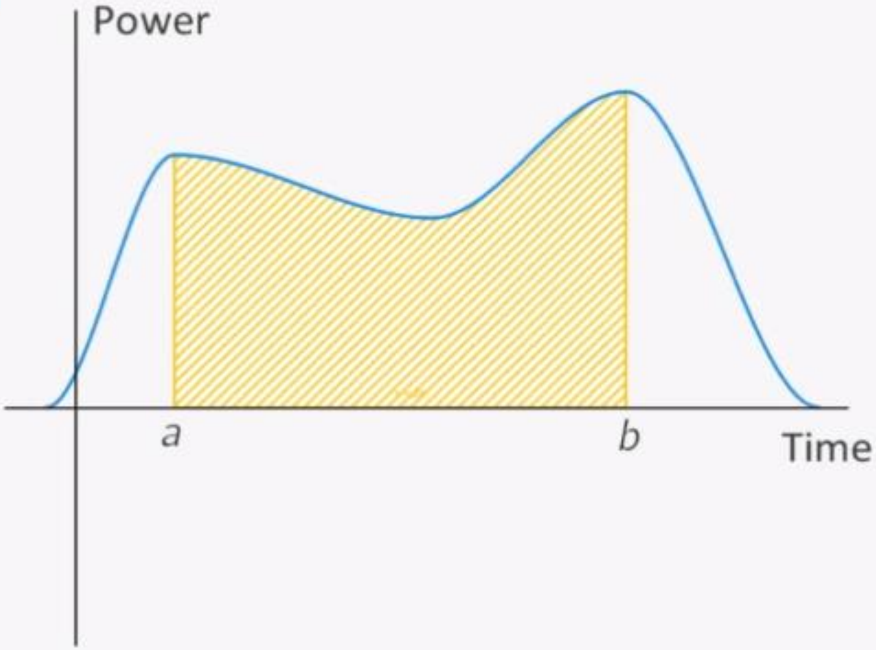
$$= \sum_{i=1}^{N} P(t_i) \cdot \Delta t \quad (N = 5)$$

Non-constant power



- Subintervals of size Δt
- Point in *each* subinterval
- Total energy
  $$= \sum_{i=1}^{N} P(t_i) \cdot \Delta t$$
- Limit: $\begin{cases} \Delta t \to 0 \\ N \to \infty \end{cases}$

Non-constant power



INTEGRATION

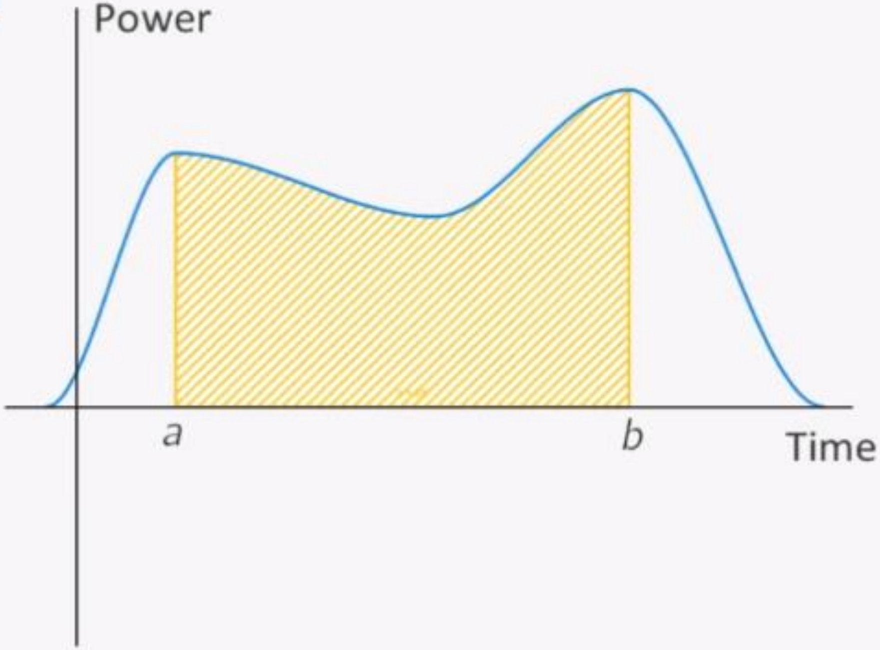
- Subintervals of size Δt
- Point in *each* subinterval
- Total energy

$$= \sum_{i=1}^{N} P(t_i) \cdot \Delta t$$

RIEMANN SUM

- Limit: $\begin{cases} \Delta t \to 0 \\ N \to \infty \end{cases}$

Non-constant power

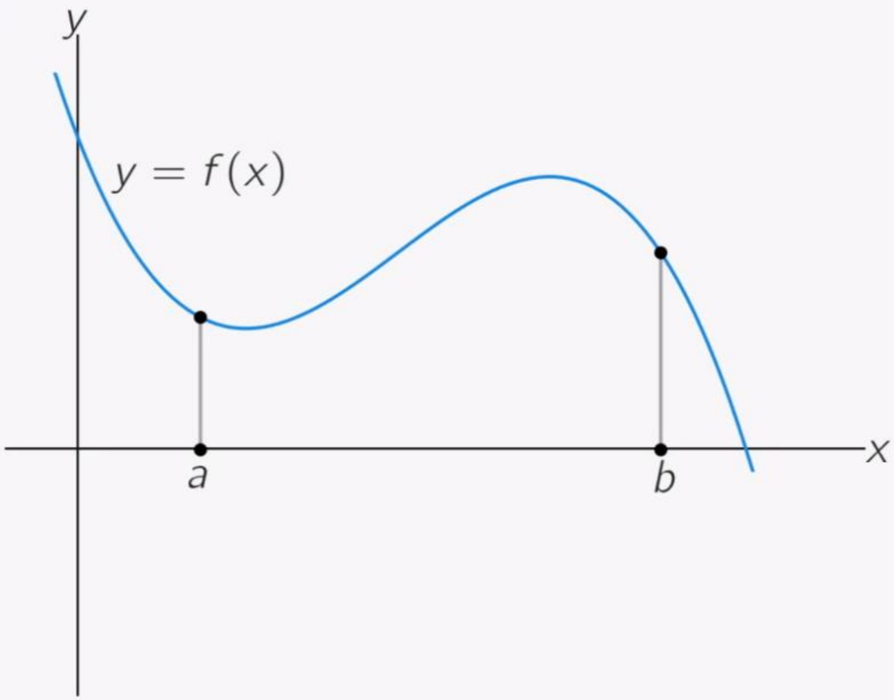


Energy consumption

= **integral**

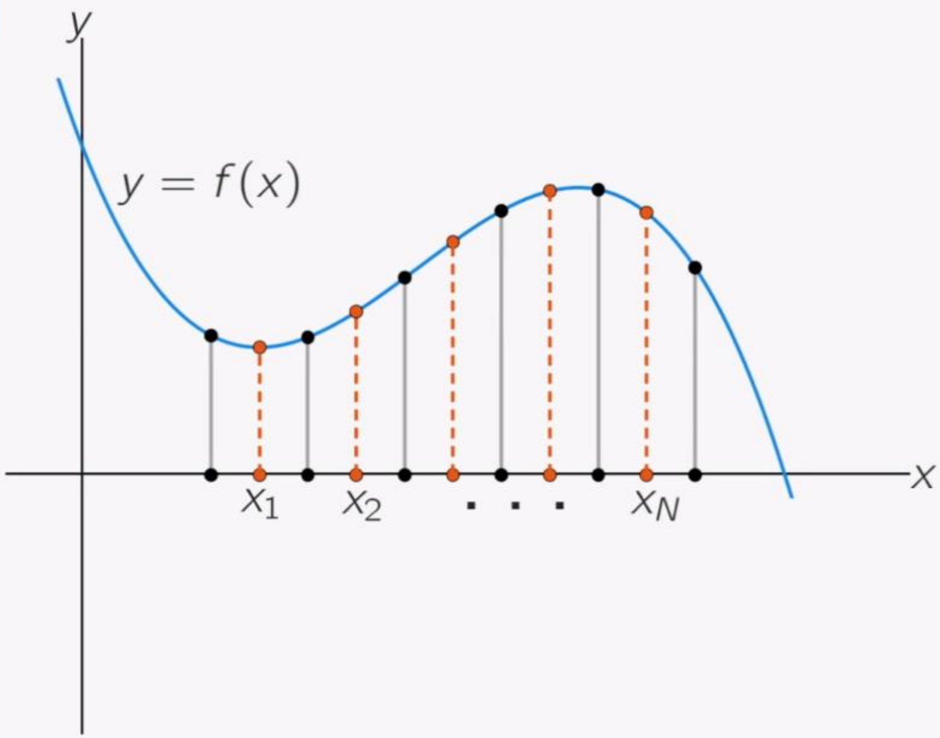
$$\int_a^b P(t)\,dt$$

Integration – the definition



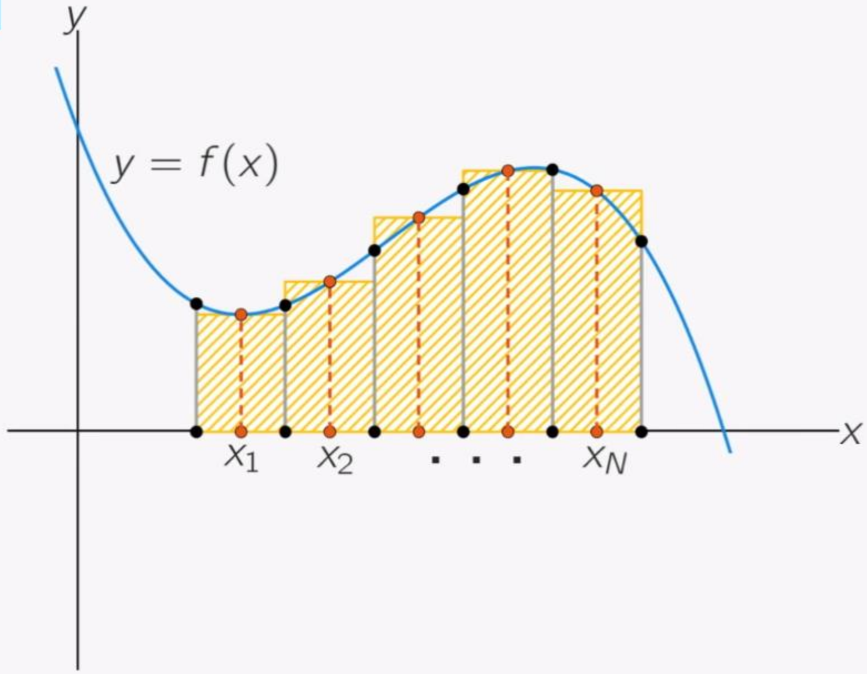
$$\int_a^b f(x)\,dx = \textbf{?}$$

Integration – the definition



- Subintervals of size Δx
- Sample points

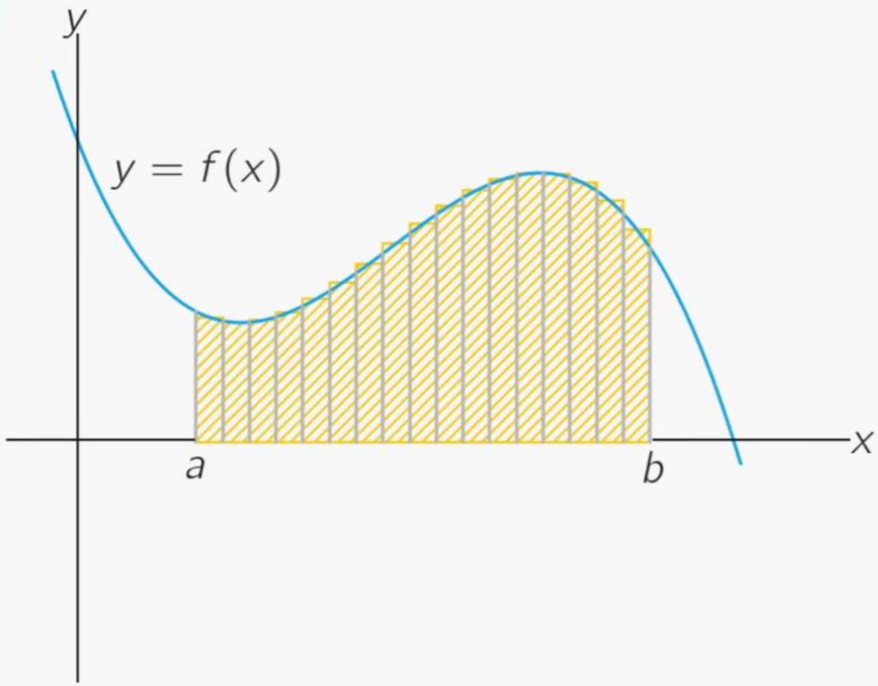
Integration – the definition



- Subintervals of size Δx
- Sample points
- Riemann sum

$$\sum_{i=1}^{N} f(x_i) \cdot \Delta x$$

Integration – the definition

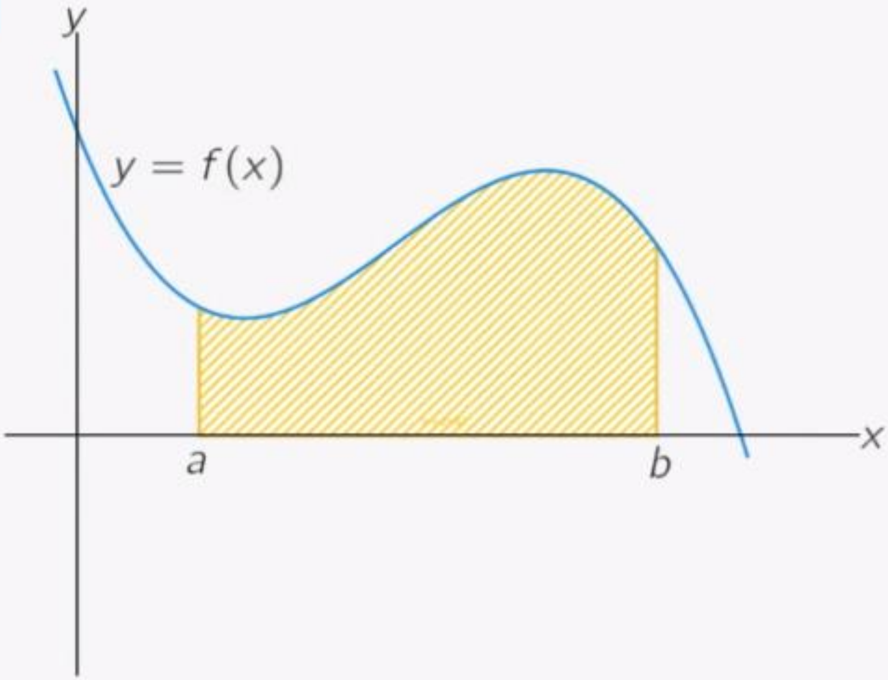


- Subintervals of size Δx
- Sample points
- Riemann sum

$$\sum_{i=1}^{N} f(x_i) \cdot \Delta x$$

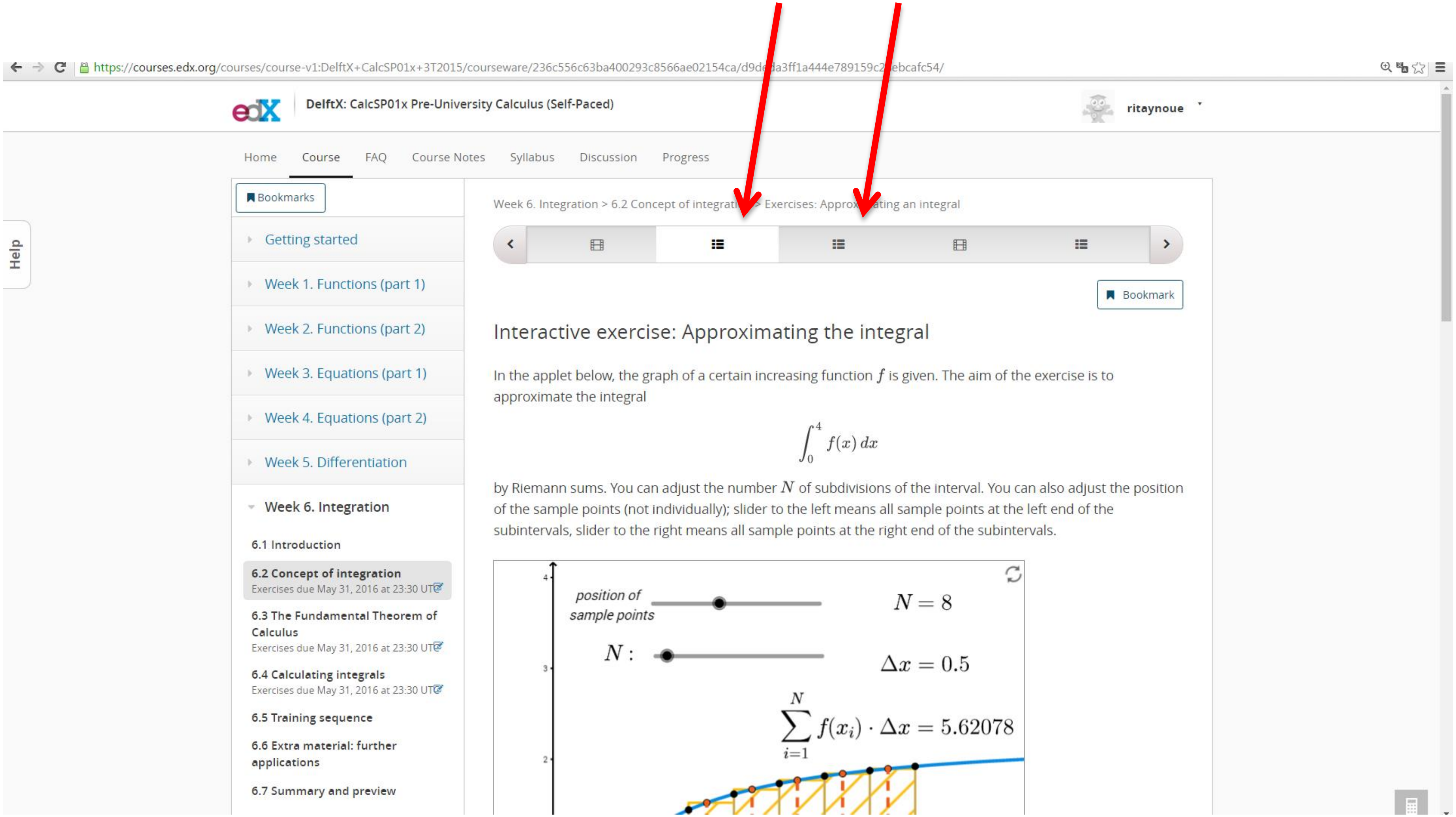
- Limit: $\begin{cases} \Delta x \to 0 \\ N \to \infty \end{cases}$

Integration – the definition



Integral:

$$\int_a^b f(x)\, dx$$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a3ff1a444e789159c2
ebcafc54/
DelftX: CalcSP01x Pre-University Calculus (Self-Paced)
ritaynoue
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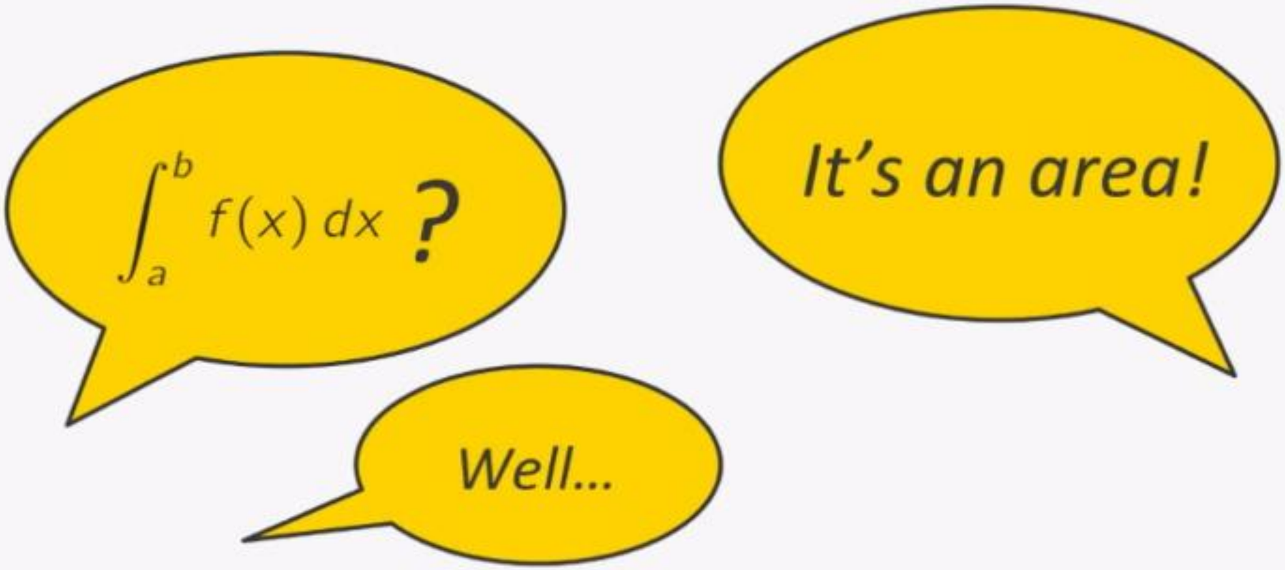
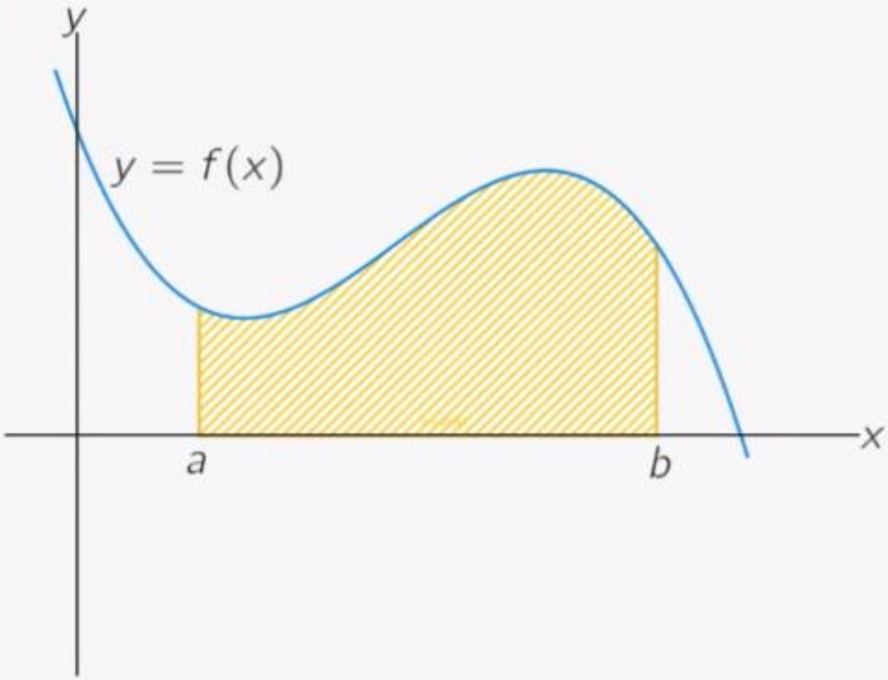
Getting started
Week 1. Functions (part 1)
Week 2. Functions (part 2)
Week 3. Equations (part 1)
Week 4. Equations (part 2)
Week 5. Differentiation
Week 6. Integration
6.1 Introduction
6.2 Concept of integration
Exercises due May 31, 2016 at 23:30 UTC
6.3 The Fundamental Theorem of Calculus
Exercises due May 31, 2016 at 23:30 UTC
6.4 Calculating integrals
Exercises due May 31, 2016 at 23:30 UTC
6.5 Training sequence
6.6 Extra material: further applications
6.7 Summary and preview
Week 6. Integration > 6.2 Concept of integrati
Exercises: Approx
ating an integral
Bookmark
Interactive exercise: Approximating the integral
In the applet below, the graph of a certain increasing function f is given. The aim of the exercise is to approximate the integral
$\int_0^4 f(x)\,dx$
by Riemann sums. You can adjust the number N of subdivisions of the interval. You can also adjust the position of the sample points (not individually); slider to the left means all sample points at the left end of the subintervals, slider to the right means all sample points at the right end of the subintervals.
position of sample points
$N = 8$
N :
$\Delta x = 0.5$
$\sum_{i=1}^{N} f(x_i) \cdot \Delta x = 5.62078$
4
3
2

The integral: what does it mean?



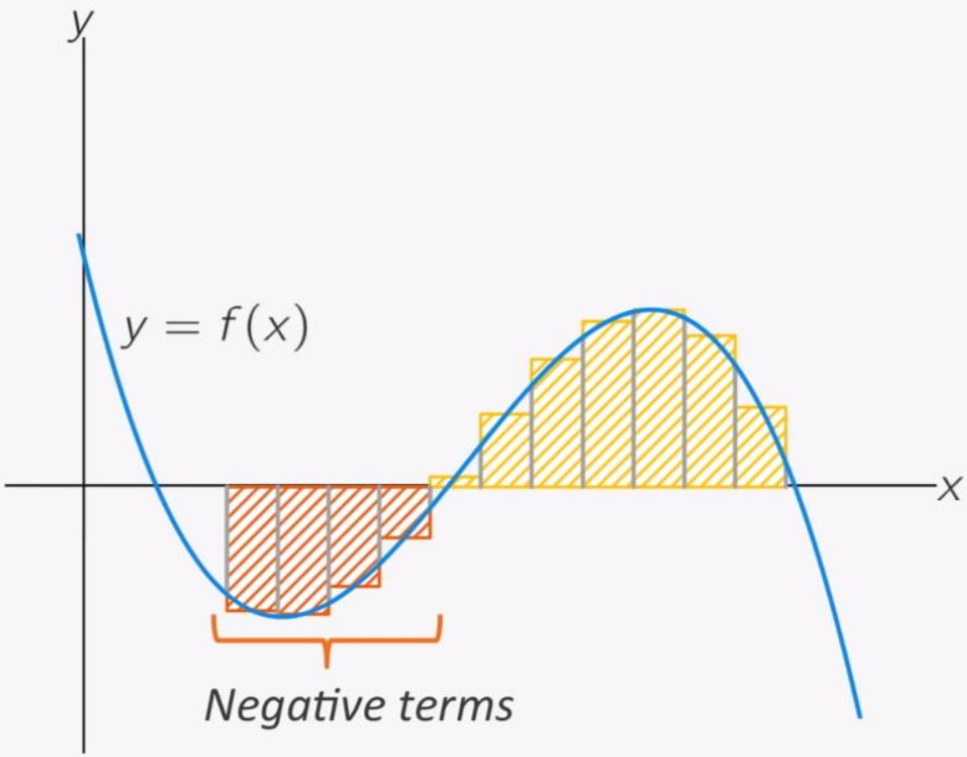
Integration – the definition



Integral:

$$\int_a^b f(x)\,dx$$

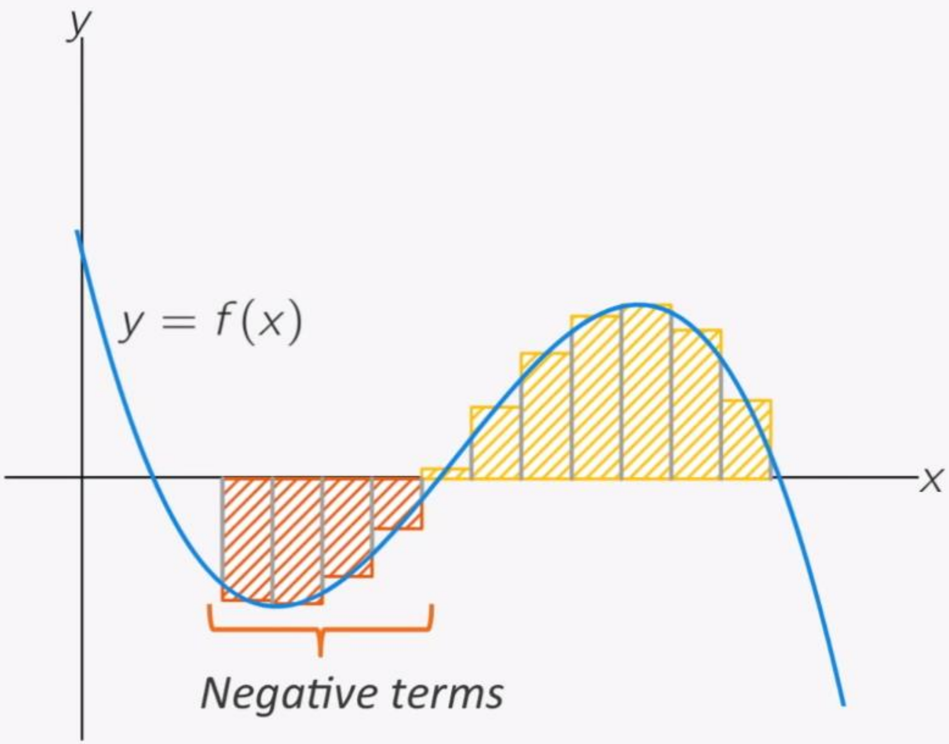
Area?



Riemann sum

$$\sum_{i=1}^{N} f(x_i) \cdot \Delta x$$

Area?



Riemann sum

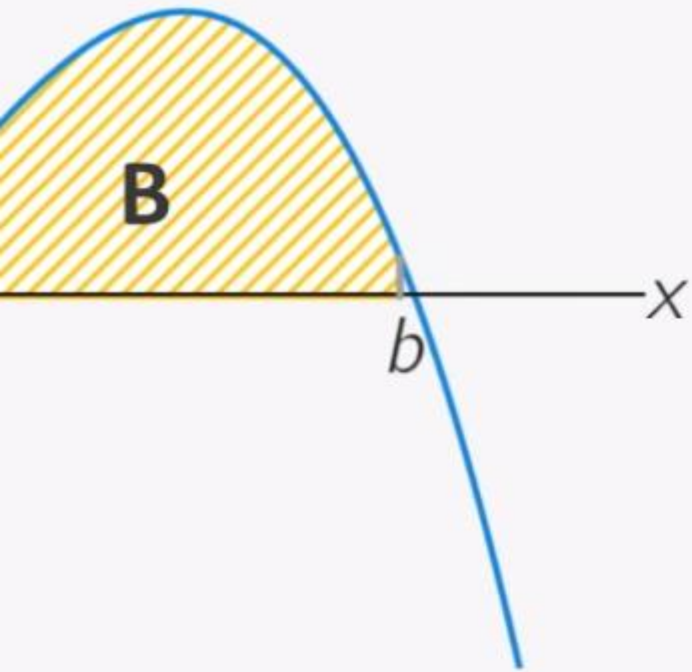
$$\sum_{i=1}^{N} f(x_i) \cdot \Delta x$$

=

area above axis

minus

area below axis



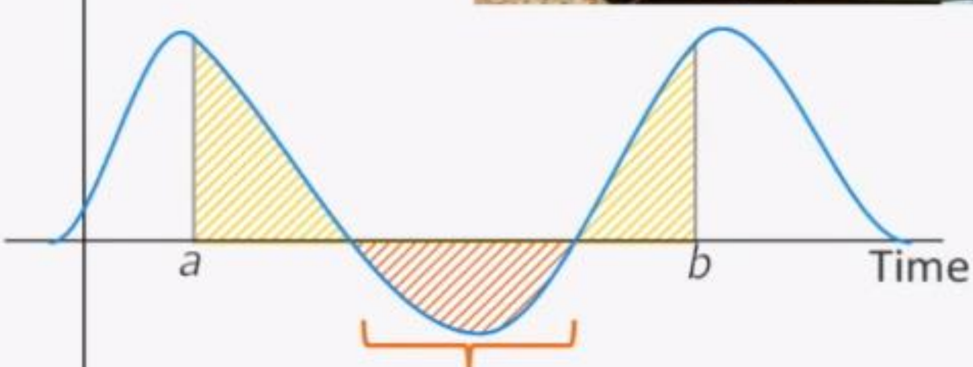
Integral

$$\int_a^b f(x)dx$$

= area B – area A

Signed area

Signed area: example

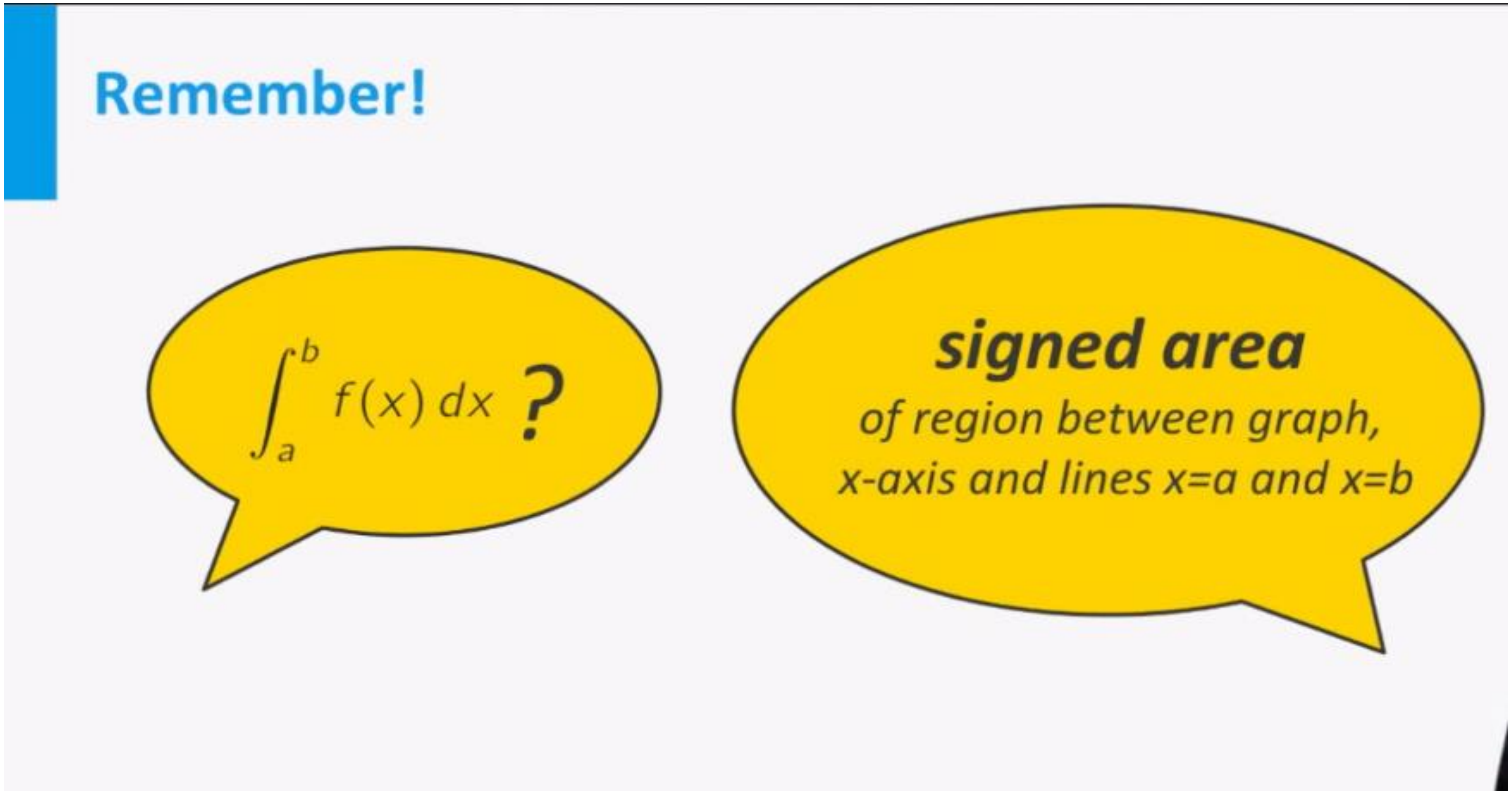


Energy consumption

$$\int_a^b P(t)\,dt$$

= signed area

Energy production = *negative* energy consumption


Remember!
$\int_a^b f(x)\,dx$?
signed area
of region between graph,
x-axis and lines x=a and x=b

https://courses.edx.org/courses/course-v1:DelftX+CalcSP01x+3T2015/courseware/236c556c63ba400293c8566ae02154ca/d9deda3ff1a444e789159c28ebcafc54/

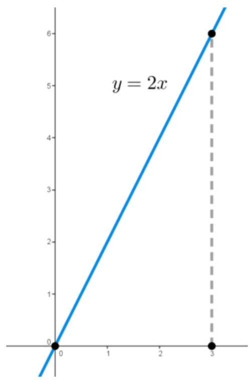
Help

Bookmark

EXERCISES: DETERMINE THE INTEGRAL GEOMETRICALLY

Exercise 4

(1 point possible)



Determine this integral geometrically.

To help you, the graph of the function is given.

\(\displaystyle \int_0^3 2x \, dx = \)

\(\)

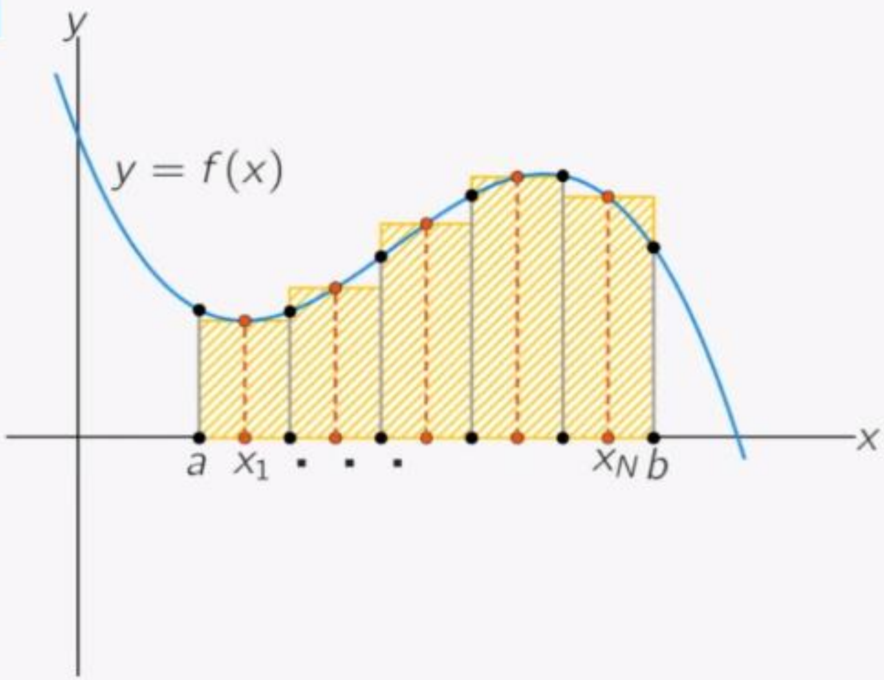
CHECK SAVE *You have used 0 of 5 submissions*

Exercise 5

(1 point possible)

Teorema Fundamental do Cálculo: Integração e diferenciação são operações inversas

The integral - approximation



$$\int_a^b f(x)\,dx$$

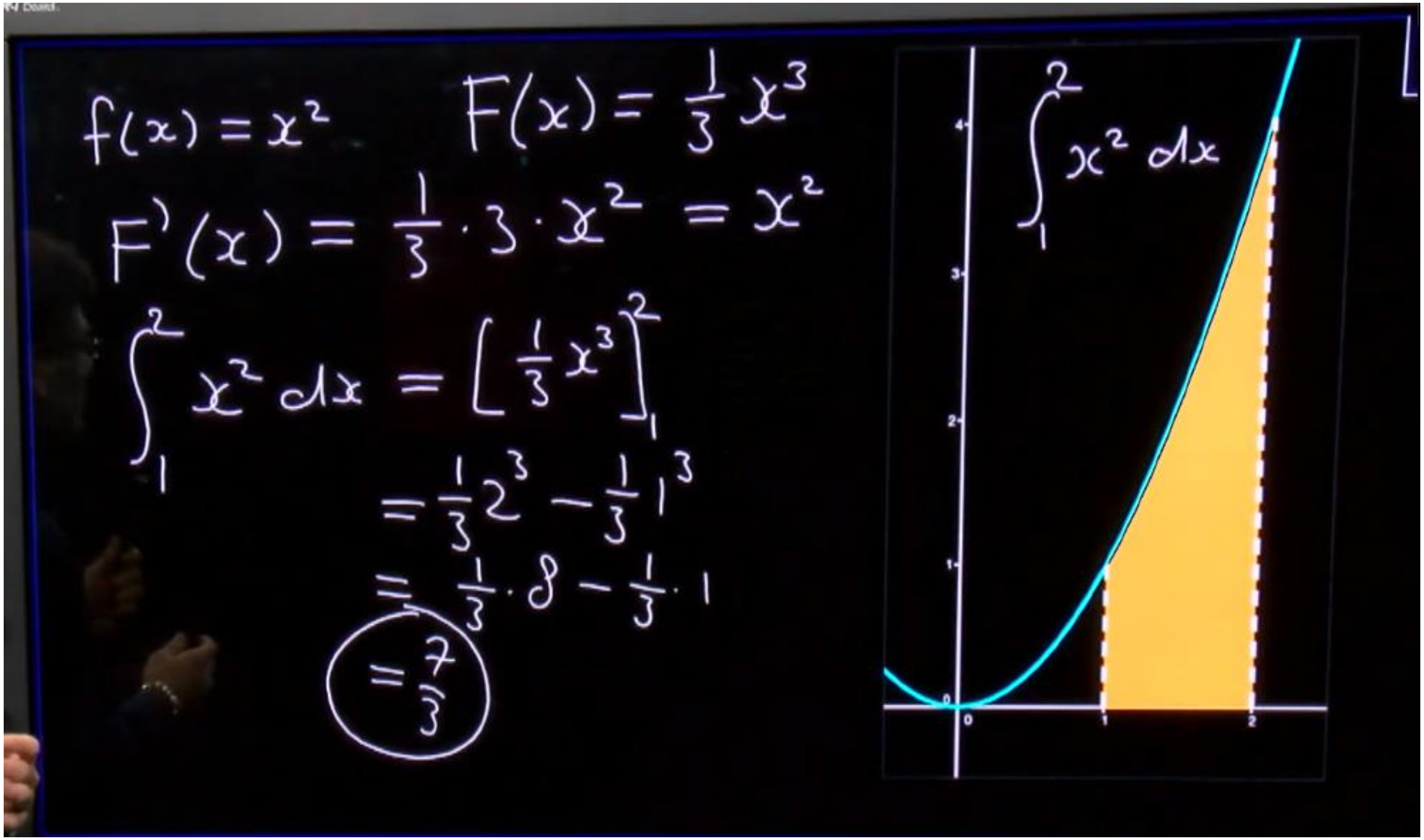
$$\approx \sum_{i=1}^{N} f(x_i)\cdot\Delta x$$

The Fundamental Theorem of Calculus

- f continuous function on $[a, b]$;
- F function such that $F' = f$;

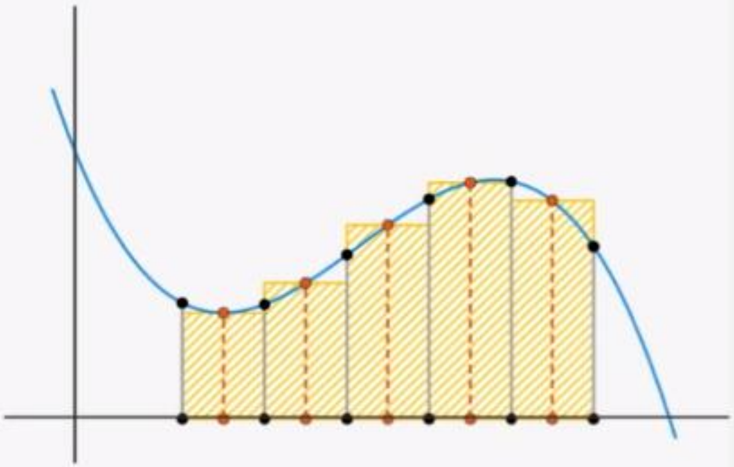
then $$\int_a^b f(x)\,dx = F(b) - F(a)$$

Primitive function or *antiderivative* of f

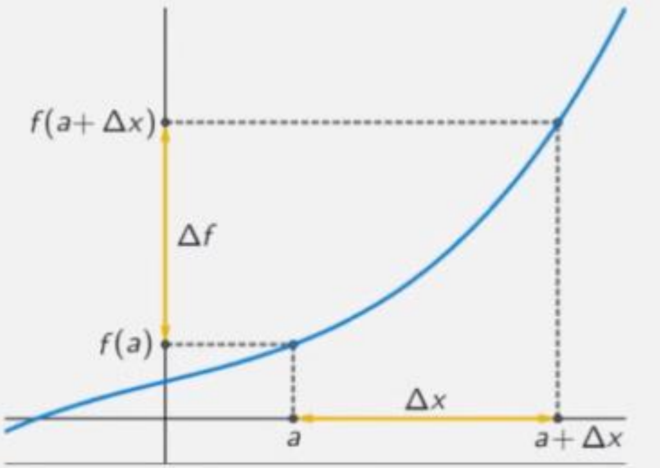

$f(x) = x^2$
$F(x) = \frac{1}{3}x^3$
$F'(x) = \frac{1}{3} \cdot 3 \cdot x^2 = x^2$
$\int_1^2 x^2 \, dx = \left[\frac{1}{3}x^3\right]_1^2$
$= \frac{1}{3}2^3 - \frac{1}{3}1^3$
$= \frac{1}{3} \cdot 8 - \frac{1}{3} \cdot 1$
$= \frac{7}{3}$
$\int_1^2 x^2 \, dx$
0
1
2
3
4

Integration vs. differentiation

$$\sum_{i=1}^{N} f(x_i) \cdot \Delta x, \begin{cases} \Delta x \to 0 \\ N \to \infty \end{cases}$$



$$\lim_{\Delta x \to 0} \frac{f(a + \Delta x) - f(a)}{\Delta x}$$



Primitive functions

Example

$$f(x) = x^2$$

Primitive functions:

$$F(x) = \frac{1}{3}x^3$$

$$G(x) = \frac{1}{3}x^3 + 10$$

$$\cdots$$

Primitive functions

Example

$f(x) = x^2$

Primitive functions:

$F(x) = \frac{1}{3}x^3$

$G(x) = \frac{1}{3}x^3 + 10$

$\dots$

General

Given function f

if F primitive

then $F + constant$ primitive

The indefinite integral

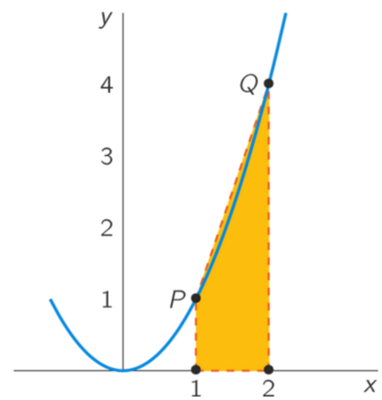
$$\int f(x)dx = \text{set of all primitive functions of } f$$

Example: $\int x^2\, dx = \frac{1}{3}x^3 + C$

Exercise 1

(1/1 point)

In the video, we saw the exact evaluation of the integral $\int_1^2 x^2\,dx$. In this case the integral represents the area of the region between the graph of x^2 and the interval $[1, 2]$ on the horizontal axis (shown in the picture). The teacher claimed that the answer $\frac{7}{3} \approx 2.33$ is reasonable, if you look at the region in the picture. Let us check this claim.



Suppose that we replace the graph between points P and Q by a straight line. Let us denote the region between this line and the interval $[1, 2]$ on the horizontal axis by R. The area of R can be determined geometrically. It provides an estimate for the area of the actual region, and hence for the integral.

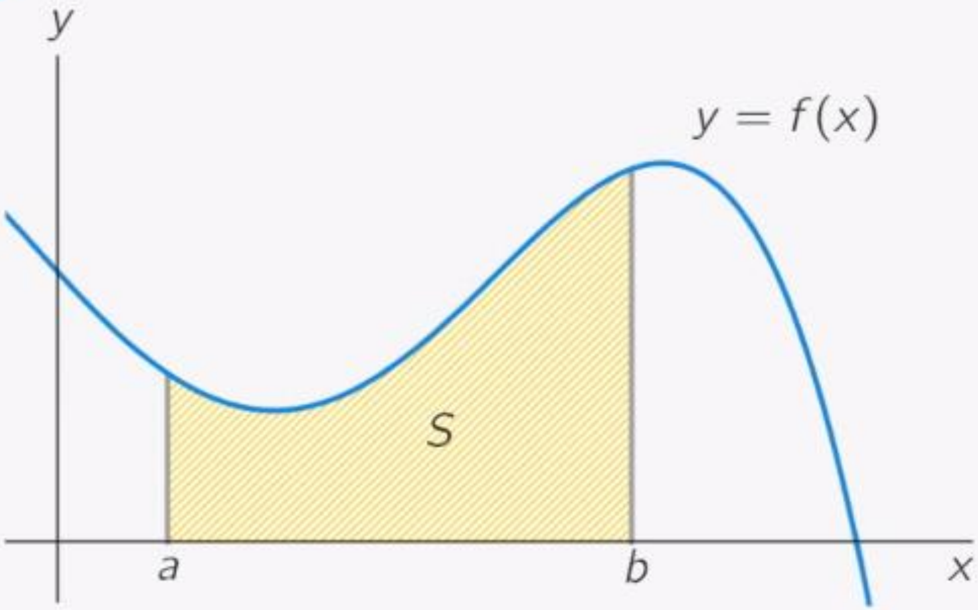
What is the area of R?

The Fundamental Theorem of Calculus

- f continuous function on $[a, b]$;
- F function such that $F' = f$;

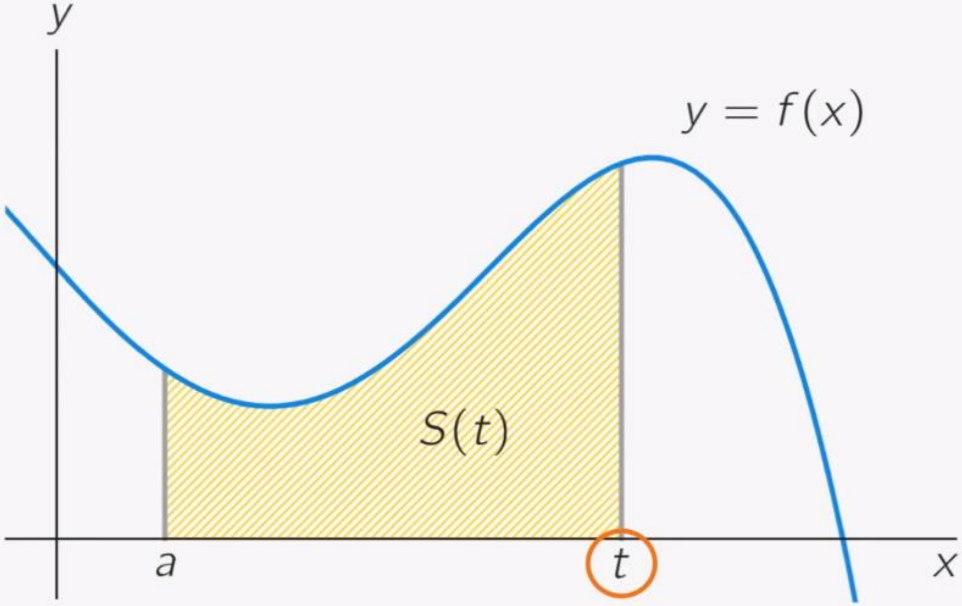
then $$\int_a^b f(x)\,dx = F(b) - F(a)$$

The signed area function



$$S = \int_a^b f(x)\,dx$$

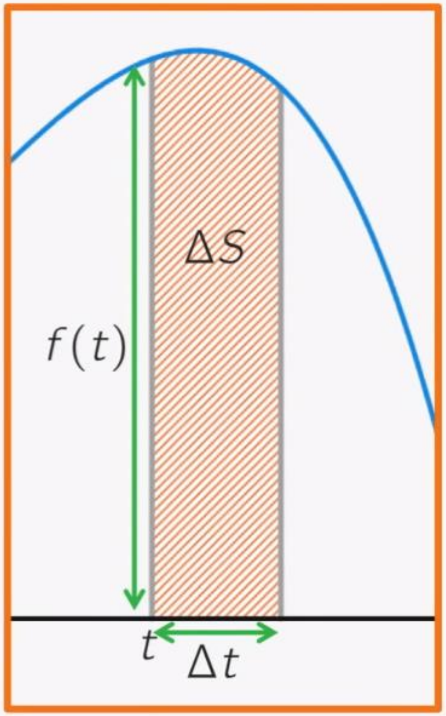
The signed area function



$$S(t) = \int_a^t f(x)\,dx$$

Claim: $S' = f$

The signed area function

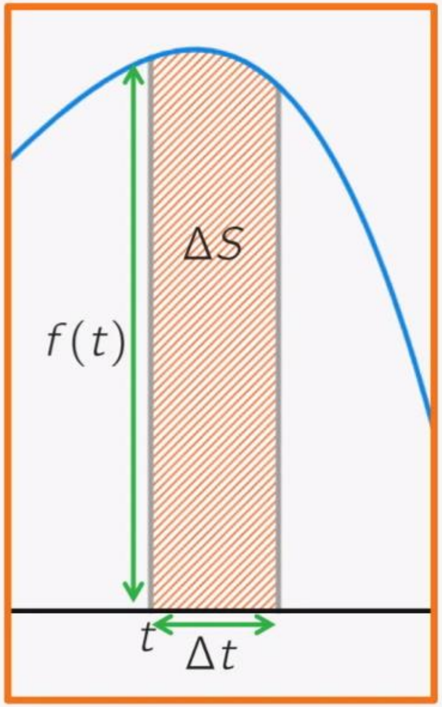


$$S(t) = \int_a^t f(x)\,dx$$

Claim: $S' = f$

$$\Delta S \approx f(t) \cdot \Delta t$$

The signed area function



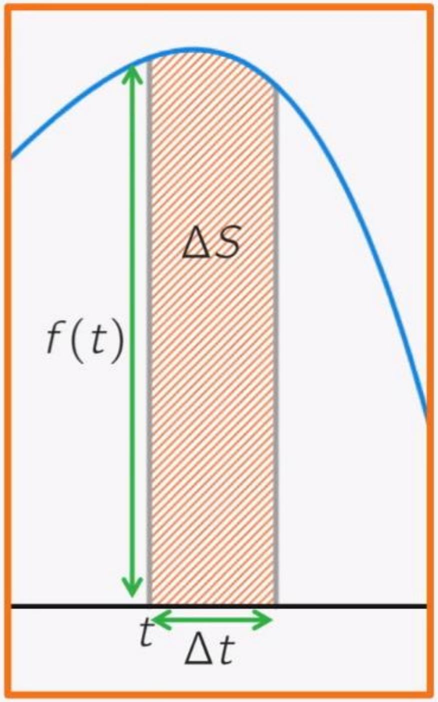
$$S(t) = \int_a^t f(x)\,dx$$

Claim: $S' = f$

$$\Delta S \approx f(t) \cdot \Delta t$$

$$\lim_{\Delta t \to 0} \frac{\Delta S}{\Delta t} = f(t)$$

The signed area function



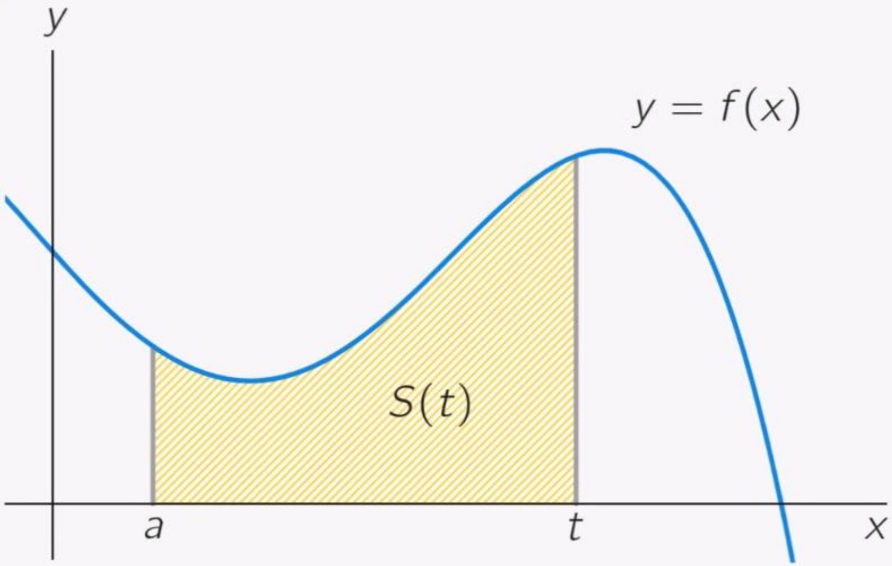
$$S(t) = \int_a^t f(x)\,dx$$

Claim: $S' = f$

$$\Delta S \approx f(t) \cdot \Delta t$$

$$S'(t) = f(t)$$

The signed area function



$$S(t) = \int_a^t f(x)\, dx$$

$$\Rightarrow \quad S' = f$$

S primitive function of f

The fundamental theorem: general case

$$S(t) = \int_a^t f(x)\,dx$$

is primitive function of f

$$\int_a^b f(x)dx = S(b) - S(a)$$

F any other primitive of f

then

$$\int_a^b f(x)dx = F(b) - F(a)$$

FUNDAMENTAL THEOREM OF CALCULUS

http://www.if.ufrgs.br/tex/fisica-4/tab-integrais.pdf

• *Derivadas*

Sejam u e v funções deriváveis de x e n constante.

1. $y = u^n \Rightarrow y' = n\,u^{n-1}u'$.
2. $y = uv \Rightarrow y' = u'v + v'u$.
3. $y = \frac{u}{v} \Rightarrow y' = \frac{u'v - v'u}{v^2}$.
4. $y = a^u \Rightarrow y' = a^u(\ln a)\,u'$, $(a > 0,\ a \neq 1)$.
5. $y = e^u \Rightarrow y' = e^u u'$.
6. $y = \log_a u \Rightarrow y' = \frac{u'}{u}\log_a e$.
7. $y = \ln u \Rightarrow y' = \frac{1}{u}u'$.
8. $y = u^v \Rightarrow y' = v\,u^{v-1}\,u' + u^v(\ln u)\,v'$.
9. $y = \text{sen}\ u \Rightarrow y' = u'\cos\ u$.
10. $y = \cos\ u \Rightarrow y' = -u'\text{sen}\ u$.
11. $y = \text{tg}\ u \Rightarrow y' = u'\sec^2 u$.
12. $y = \text{cotg}\ u \Rightarrow y' = -u'\text{cosec}^2 u$.
13. $y = \sec\ u \Rightarrow y' = u'\sec\ u\ \text{tg}\ u$.
14. $y = \text{cosec}\ u \Rightarrow y' = -u'\text{cosec}\ u\ \text{cotg}\ u$.
15. $y = arc\ \text{sen}\ u \Rightarrow y' = \frac{u'}{\sqrt{1-u^2}}$.
16. $y = arc\ \cos\ u \Rightarrow y' = \frac{-u'}{\sqrt{1-u^2}}$.
17. $y = arc\ \text{tg}\ u \Rightarrow y' = \frac{u'}{1+u^2}$.
18. $y = arc\ \cot g\ u \Rightarrow \frac{-u'}{1+u^2}$.
19. $y = arc\ \sec\ u$, $|u| \geqslant 1$
    $\Rightarrow y' = \frac{u'}{|u|\sqrt{u^2-1}}$, $|u| > 1$.
20. $y = arc\ \text{cosec}\ u$, $|u| \geqslant 1$
    $\Rightarrow y' = \frac{-u'}{|u|\sqrt{u^2-1}}$, $|u| > 1$.

• *Integrais*

1. $\int du = u + c$.
2. $\int u^n du = \frac{u^{n+1}}{n+1} + c,\ n \neq -1$.
3. $\int \frac{du}{u} = \ln|u| + c$.
4. $\int a^u du = \frac{a^u}{\ln\ a} + c,\ a > 0,\ a \neq 1$.
5. $\int e^u du = e^u + c$.
6. $\int \text{sen}\ u\ du = -\cos\ u + c$.
7. $\int \cos\ u\ du = \text{sen}\ u + c$.
8. $\int \text{tg}\ u\ du = \ln|\sec\ u| + c$.
9. $\int \text{cotg}\ u\ du = \ln|\text{sen}\ u| + c$.
10. $\int \sec\ u\ du = \ln|\sec\ u + \text{tg}\ u| + c$.
11. $\int \text{cosec}\ u\ du = \ln|\text{cosec}\ u - \text{cotg}\ u| + c$.
12. $\int \sec\ u\ \text{tg}\ u\ du = \sec\ u + c$.
13. $\int \text{cosec}\ u\ \text{cotg}\ u\ du = -\text{cosec}\ u + c$.
14. $\int \sec^2 u\ du = \text{tg}\ u + c$.
15. $\int \text{cosec}^2 u\ du = -\text{cotg}\ u + c$.
16. $\int \frac{du}{u^2+a^2} = \frac{1}{a} arc\ \text{tg}\frac{u}{a} + c$.
17. $\int \frac{du}{u^2-a^2} = \frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right| + c,\ u^2 > a^2$.
18. $\int \frac{du}{\sqrt{u^2+a^2}} = \ln\left|u + \sqrt{u^2+a^2}\right| + c$.
19. $\int \frac{du}{\sqrt{u^2-a^2}} = \ln\left|u + \sqrt{u^2-a^2}\right| + c$.
20. $\int \frac{du}{\sqrt{a^2-u^2}} = arc\ \text{sen}\frac{u}{a} + c,\ u^2 < a^2$.
21. $\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a} arc\ \sec\left|\frac{u}{a}\right| + c$.

http://www.if.ufrgs.br/tex/fisica-4/tab-integrais.pdf

• *Identidades Trigonométricas*

1. $\text{sen}^2 x + \cos^2 x = 1.$
2. $1 + \text{tg}^2 x = \sec^2 x.$
3. $1 + \text{cotg}^2 x = \text{cosec}^2 x.$
4. $\text{sen}^2 x = \frac{1-\cos\ 2x}{2}.$
5. $\cos^2 x = \frac{1+\cos\ 2x}{2}.$
6. $\text{sen}\ 2x = 2\ \text{sen}\ x\ \cos\ x.$
7. $2\ \text{sen}\ x\ \cos\ y = \text{sen}\,(x-y) + sen\,(x+y).$
8. $2\ \text{sen}\ x\ \text{sen}\ y = \cos\,(x-y) - \cos\,(x+y).$
9. $2\ \cos\ x\ \cos\ y = \cos\,(x-y) + \cos\,(x+y).$
10. $1 \pm \text{sen}\ x = 1 \pm \cos\left(\frac{\pi}{2} - x\right).$

• *Fórmulas de Recorrência*

1. $\int sen^n au\, du = -\frac{sen^{n-1} au\ \cos\ au}{an} + \left(\frac{n-1}{n}\right) \int sen^{n-2} au\, du.$
2. $\int \cos^n au\, du = \frac{sen\ au\ \cos^{n-1} au}{an} + \left(\frac{n-1}{n}\right) \int \cos^{n-2} au\, du.$
3. $\int tg^n au\, du = \frac{tg^{n-1} au}{a(n-1)} - \int tg^{n-2} au\, du.$
4. $\int cotg^n au\, du = -\frac{cotg^{n-1} au}{a(n-1)} - \int cotg^{n-2} au\, du.$
5. $\int \sec^n au\, du = \frac{\sec^{n-2} au\ tg\ au}{a(n-1)} + \left(\frac{n-2}{n-1}\right) \int \sec^{n-2} au\, du.$
6. $\int cosec^n au\, du = -\frac{cosec^{n-2} au\ cotg\ au}{a(n-1)} + \left(\frac{n-2}{n-1}\right) \int cosec^{n-2} au\, du.$