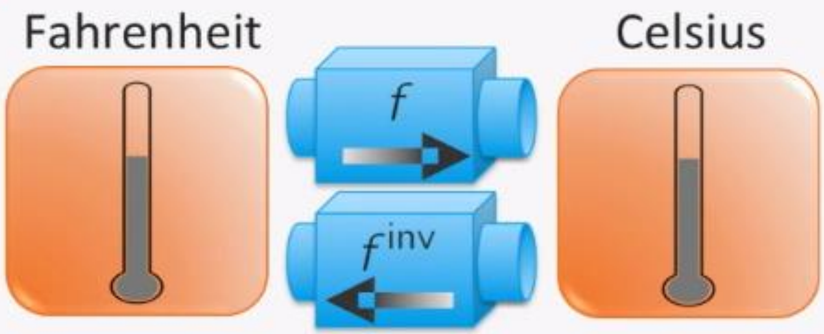
Funções Inversas e Logarítmicas

DelftX: CalcSP01x Pre-University Calculus (Self-Paced)

2.5 Inverses and logarithms

Celsius vs Fahrenheit



F	C
32	0
41	5
50	10
59	15
68	20
77	25

- The domain of f is the range of f^{inv}
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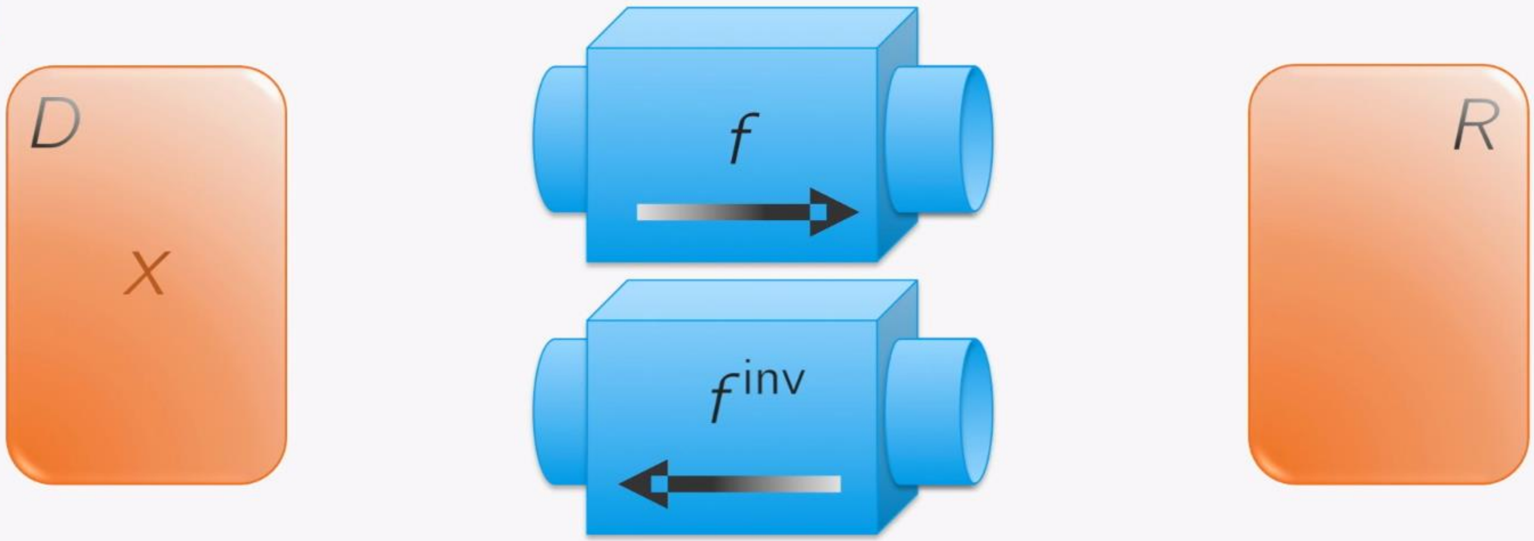
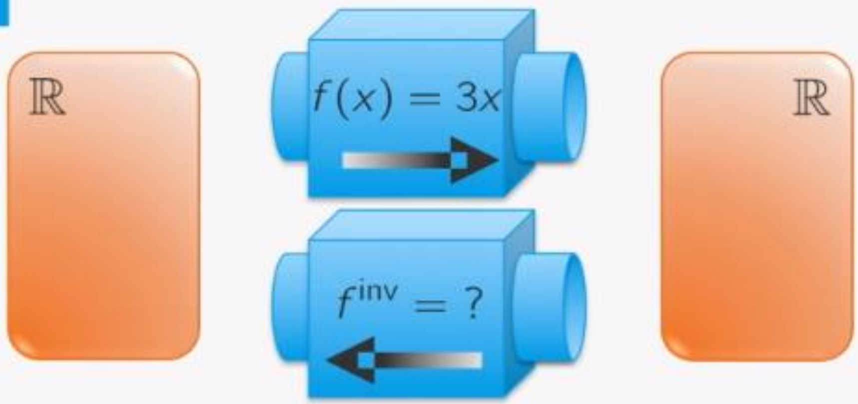
The inverse function



$$y = f(x) \longleftrightarrow f^{\text{inv}}(y) = x$$

Example: $f(x)=3x$

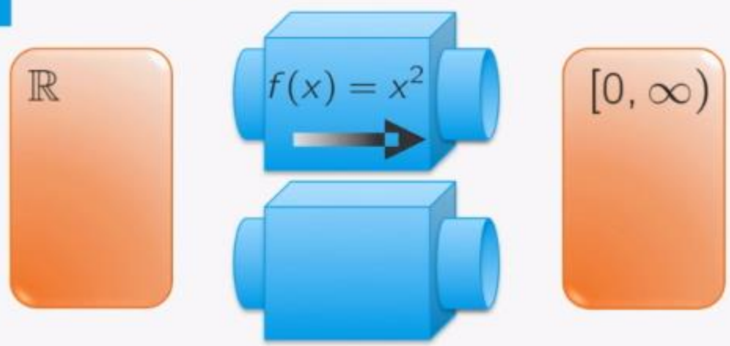


x	y
-2	-6
-1	-3
0	0
1	3
2	6

$$y = 3x \longrightarrow x = \tfrac{1}{3}y$$

$$\longrightarrow f^{\text{inv}}(y) = \tfrac{1}{3}y$$

Example: $f(x)=x^2$



x	y
-2	4
-1	1
0	0
1	1
2	4

$$y = x^2 \longrightarrow x = \sqrt{y} \text{ or } x = -\sqrt{y}$$

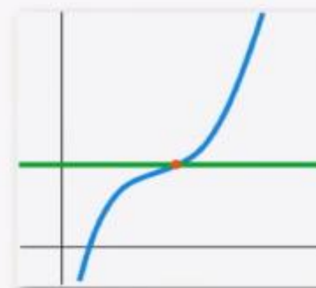
x^2 with domain $\mathbb{R}$ has no inverse function!

Horizontal line test

f has an inverse ⟷ For each y in the range of f the equation $y = f(x)$ has exactly one solution.

f is injective

Horizontal line test
f is injective if the graph of f intersects any horizontal line in at most one point

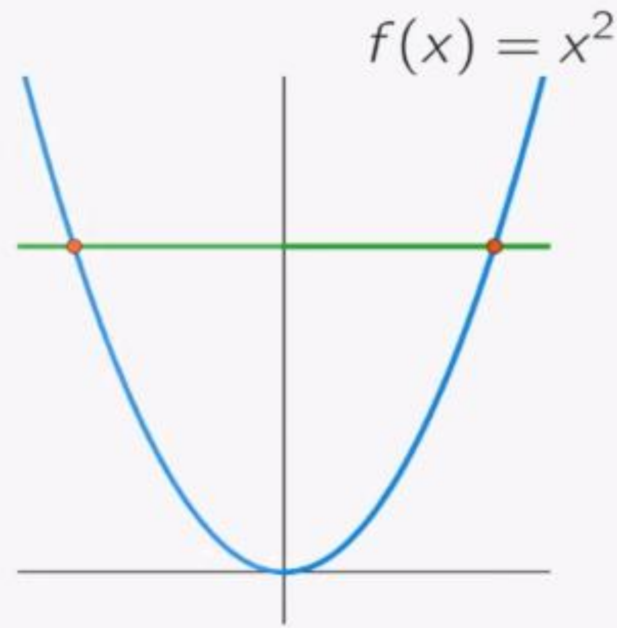


Example: $f(x)=x^2$ again

x	y
-2	4
-1	1
0	0
1	1
2	4

$y = x^2 \longrightarrow$

$x = \sqrt{y}$ or $x = -\sqrt{y}$

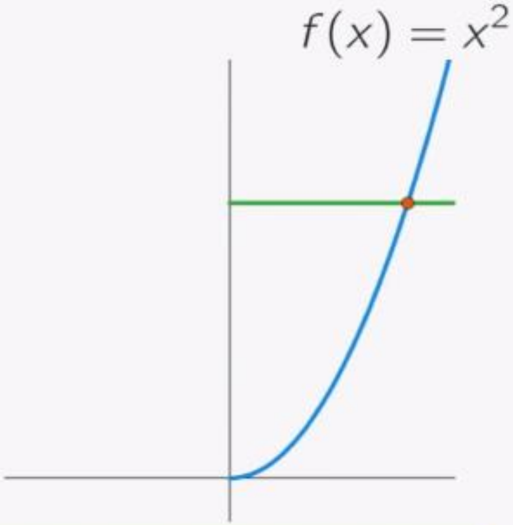


Example: $f(x)=x^2$ again

x	y
0	0
1	1
2	4

$y = x^2 \longrightarrow$

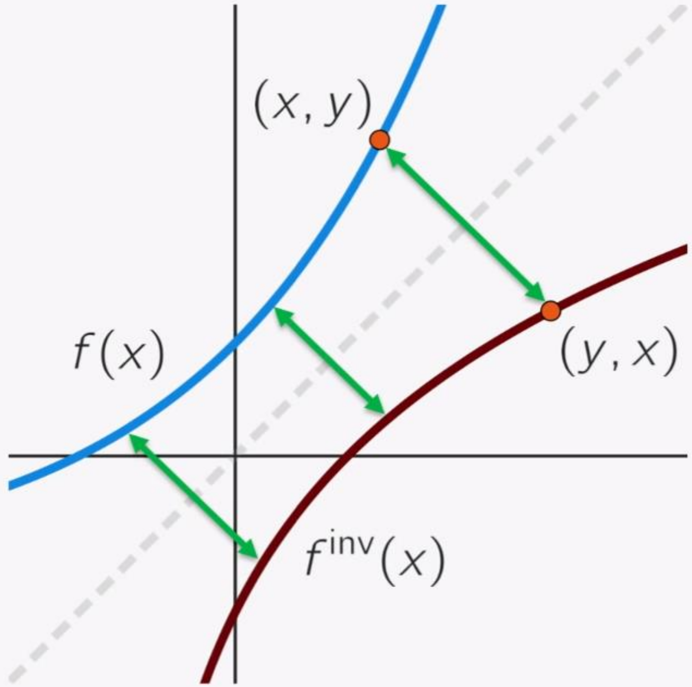
$x = \sqrt{y}$ or $x = -\sqrt{y}$ ✖



$f(x) = x^2$ on $[0, \infty)$

$f^{\text{inv}}(x) = \sqrt{x}$

The graph of an inverse function

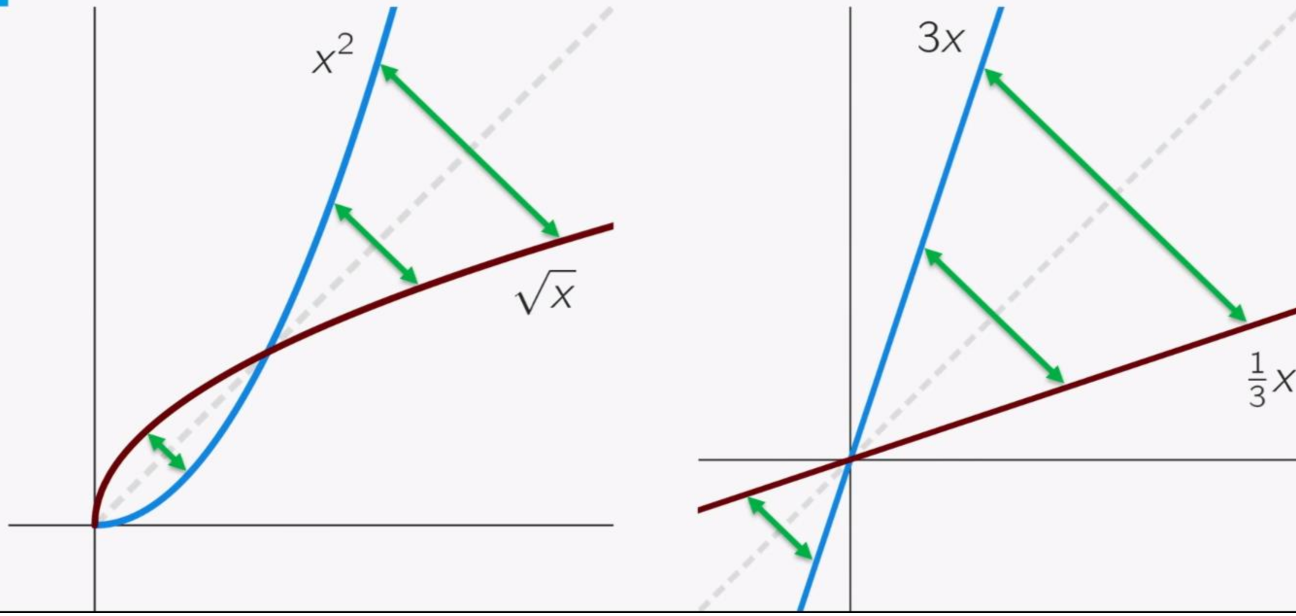


$$y = f(x)$$

$$\updownarrow$$

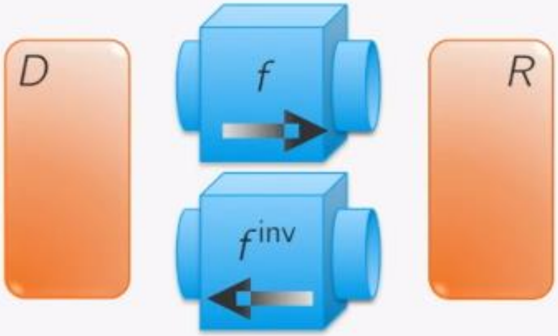
$$x = f^{\text{inv}}(y)$$

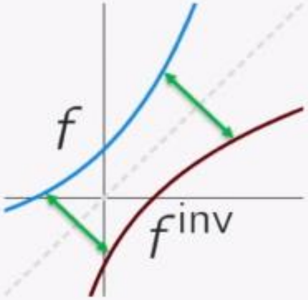
The graph of an inverse function



Summary

- Inverse exists $\longleftrightarrow$ injective
  - ▸ Horizontal line test
- $f^{\text{inv}}(y) = x \longleftrightarrow y = f(x)$
- f^{inv} has domain R and range D
- Graph: reflect in $y = x$





Combinations of 0's and 1's

#boxes		#combinations
1	0, 1	$2 = 2^1$
2	00, 01, 10, 11	$4 = 2^2$
3	000, 001, 010, 011, 100, 101, 110, 111	$8 = 2^3$
...		...
x		2^x

Combinations of 0's and 1's

At least 1000 combinations: how many boxes?

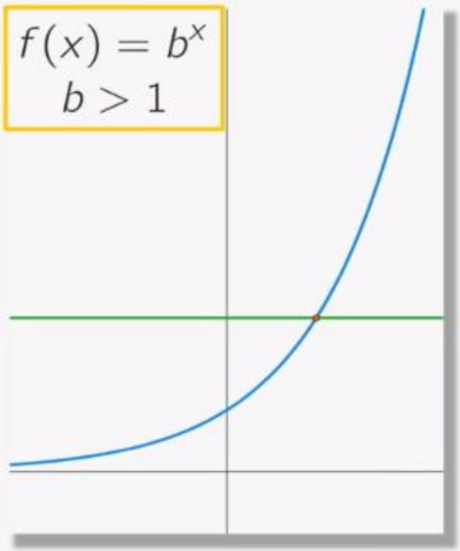
Solve: $2^x = 1000$

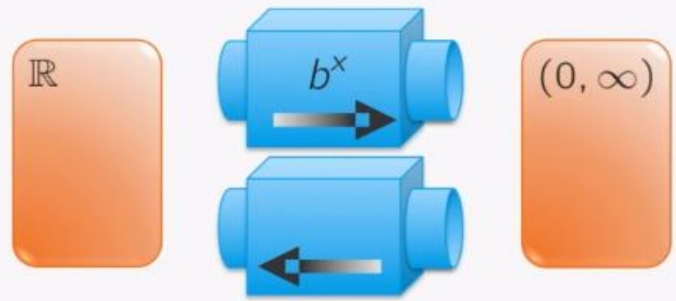
$$\left.\begin{aligned} 2^9 &= 512 \\ 2^{10} &= 1024 \end{aligned}\right\} \; 9 < x < 10 \longrightarrow 10 \text{ boxes}$$

At least 10 billion combinations: how many boxes?

Solve: $2^x = 10$ billion.

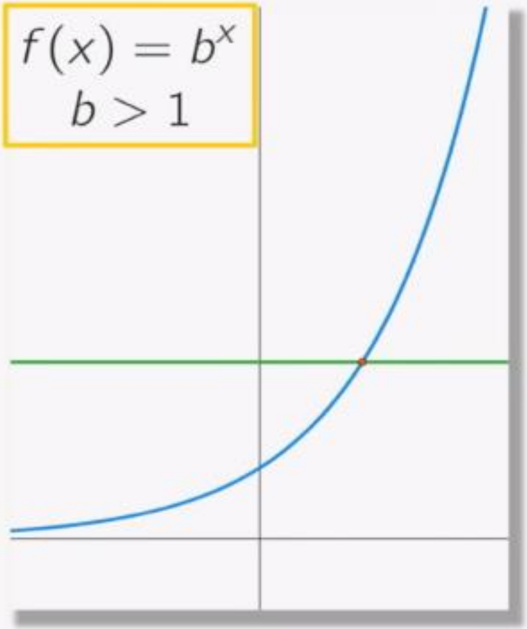
The exponential function



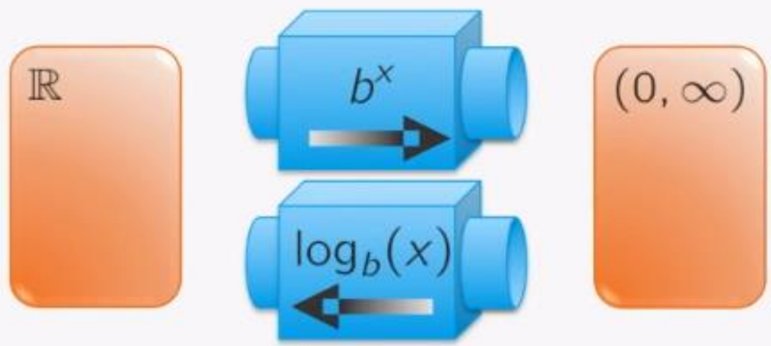


The exponential function

$$f(x) = b^x$$
$$b > 1$$



$$f^{\text{inv}}(x) = \log_b(x)$$



The logarithm

$\log_b(x)$ is the inverse of b^x

- $y = \log_b(x) \longleftrightarrow x = b^y$
- Domain: $(0, \infty)$
- Range: $\mathbb{R}$

- $\log_2(8) = \log_2(2^3) = 3$
- $\log_5(\frac{1}{5}) = \log_5(5^{-1}) = -1$
- $\log_b(b^a) = a$
- $\log_b(1) = \log(b^0) = 0$

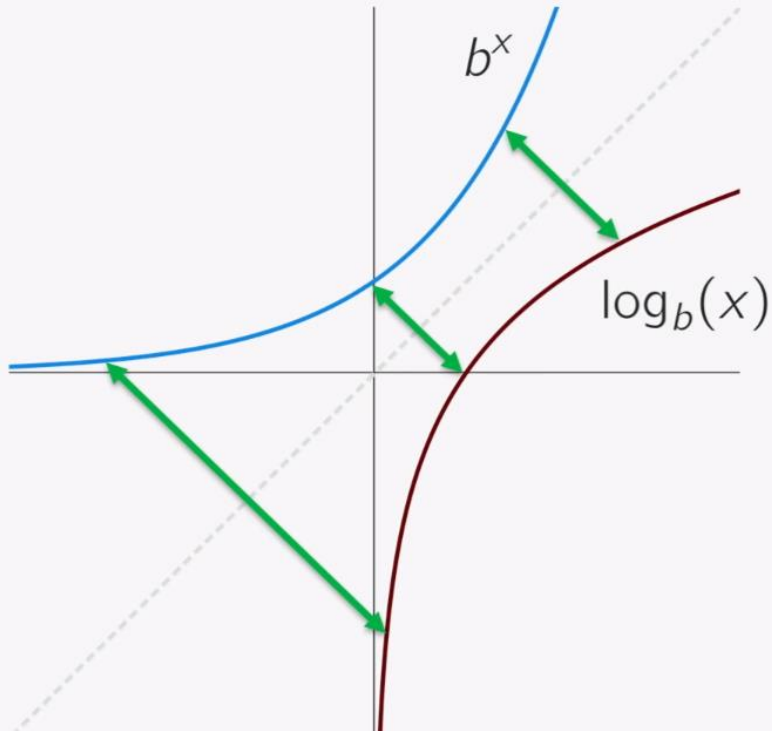
Special bases

Common bases b for logarithms:

- $b = 10$
- $b = e = 2.71828\ldots$
- $b = 2$

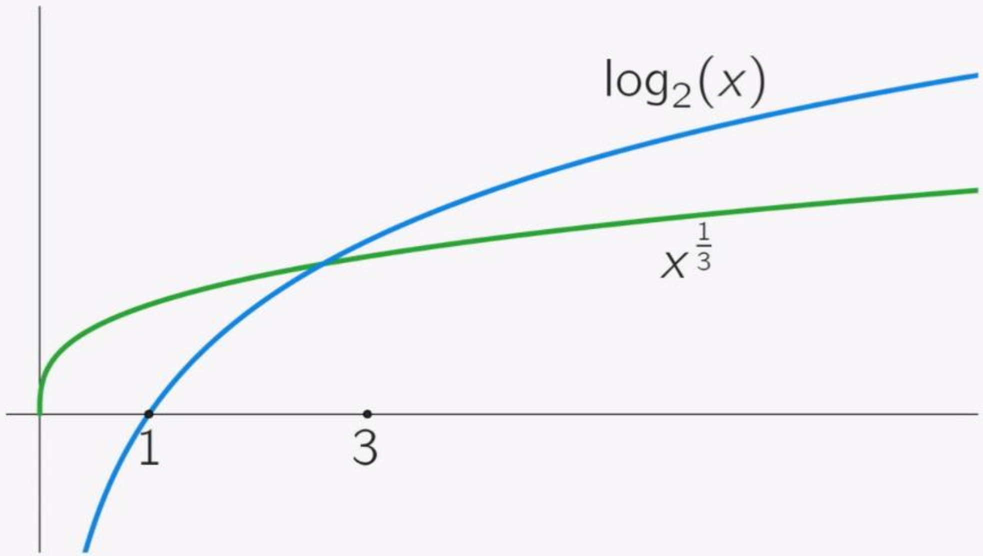
- The natural logarithm
- $\log_e(x) = \ln(x)$

The graph of $\log_b(x)$



Comparison with x^d

b^x grows faster than x^c ⟶ $\log_b(x)$ grows slower than $x^{\frac{1}{c}} = x^d$



Comparison with x^d

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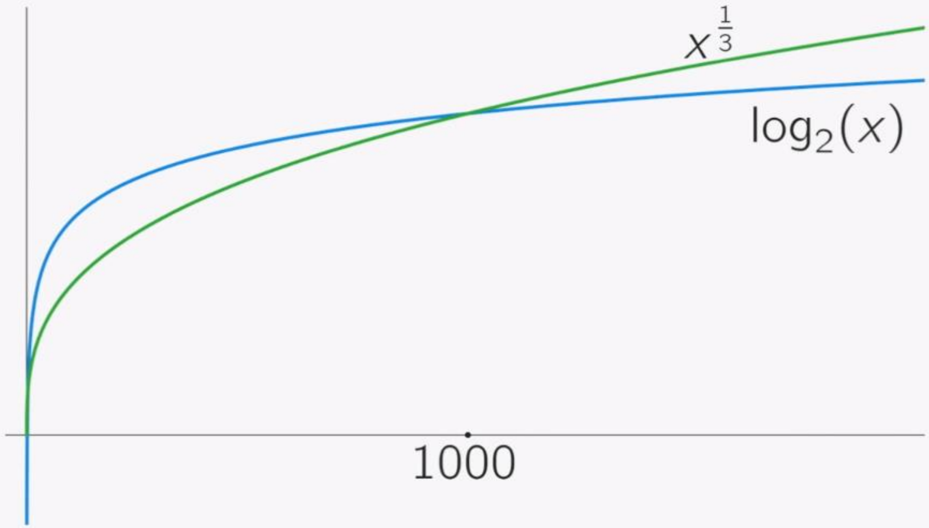

Summary

- $y = \log_b(x) \longleftrightarrow b^y = x$
- $\log_b(x)$ grows slower than x^d for any $d > 0$

Rules of calculation for logarithms

- $\log_b(AB) = \log_b(A) + \log_b(B)$
- $\log_b\left(\frac{A}{B}\right) = \log_b(A) - \log_b(B)$
- $\log_b(A^K) = K \log_b(A)$
- $\log_b(A) = \frac{\log_c(A)}{\log_c(b)}$