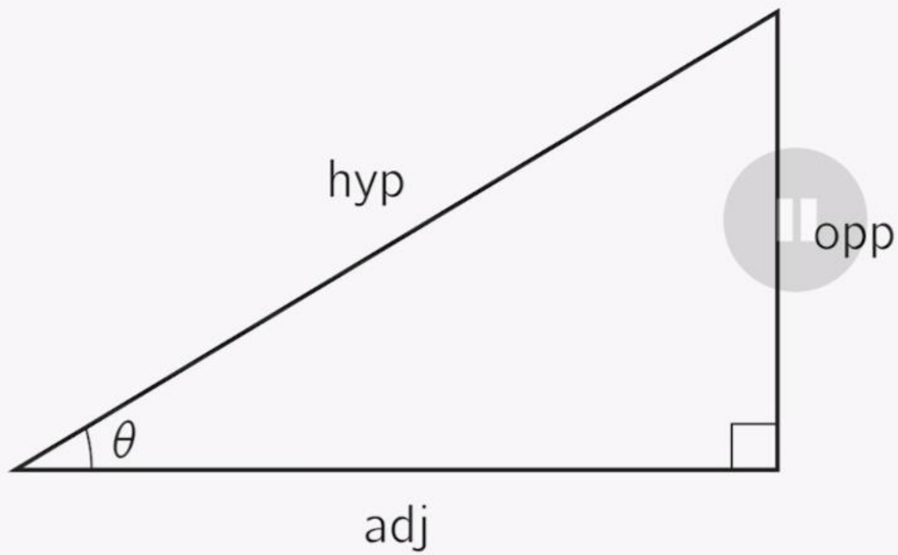
Funções Trigonométricas

DelftX: CalcSP01x Pre-University Calculus (Self-Paced)

2.2 Trigonometric functions

Trigonometric functions



$$\sin(\theta) = \frac{\text{opp}}{\text{hyp}}$$

$$\cos(\theta) = \frac{\text{adj}}{\text{hyp}}$$

$$\tan(\theta) = \frac{\text{opp}}{\text{adj}} = \frac{\sin(\theta)}{\cos(\theta)}$$

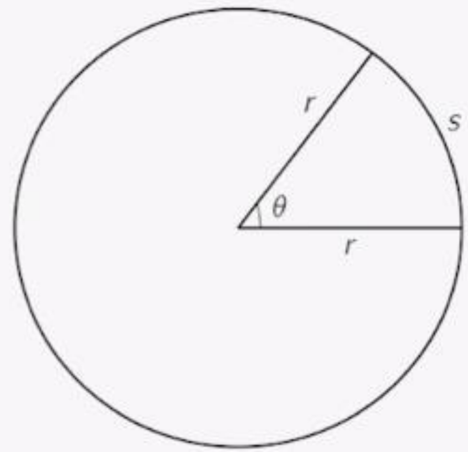
Trigonometric functions – radians vs. degrees

$$2\,\pi \text{ rad} = 360^\circ$$

$$\pi \text{ rad} = 180^\circ$$

$$1 \text{ rad} = \left(\frac{180}{\pi}\right)^\circ$$

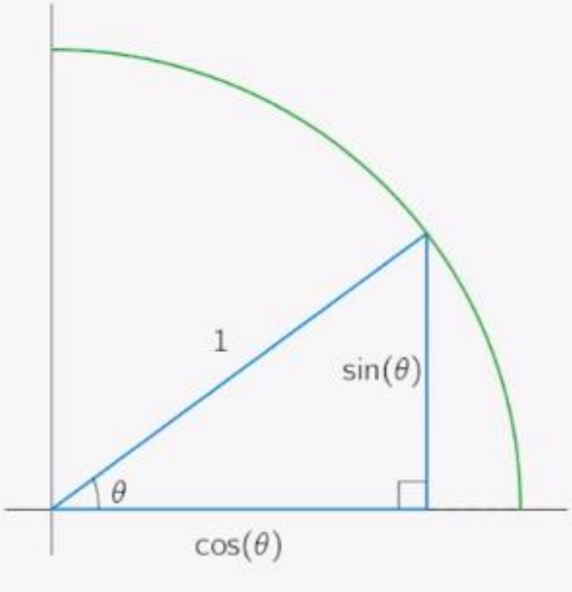
$$1^\circ = \frac{\pi}{180} \text{ rad}$$

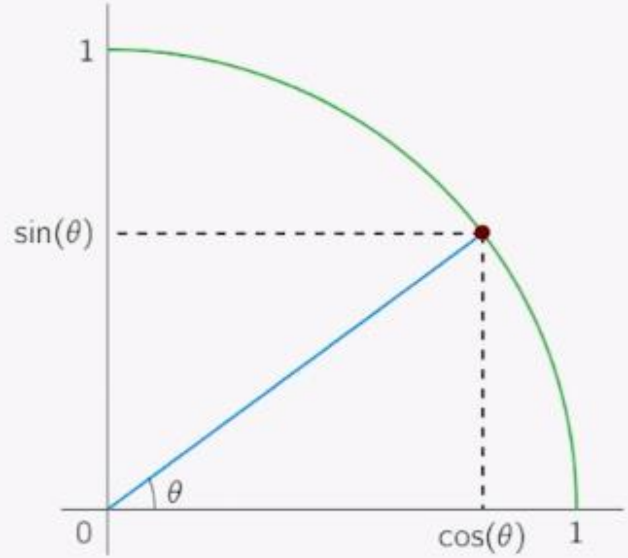


$$s = \frac{\theta}{2\pi} \cdot 2\pi\, r = r \cdot \theta$$

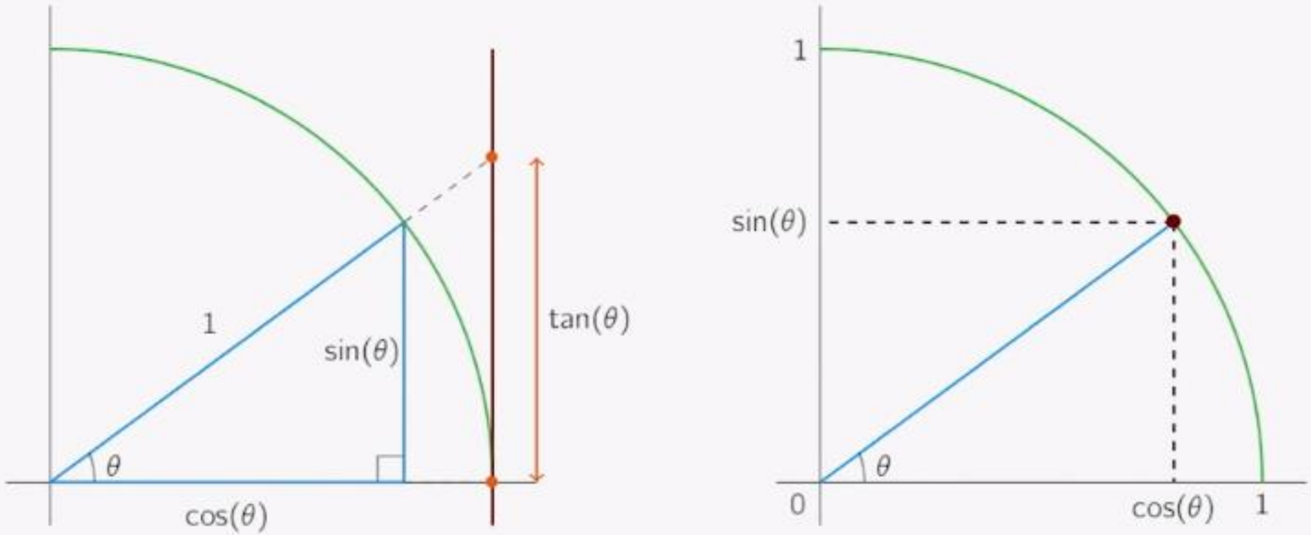
$$r = 1$$
$$\downarrow$$
$$s = \theta$$

Trigonometric functions – the unit circle

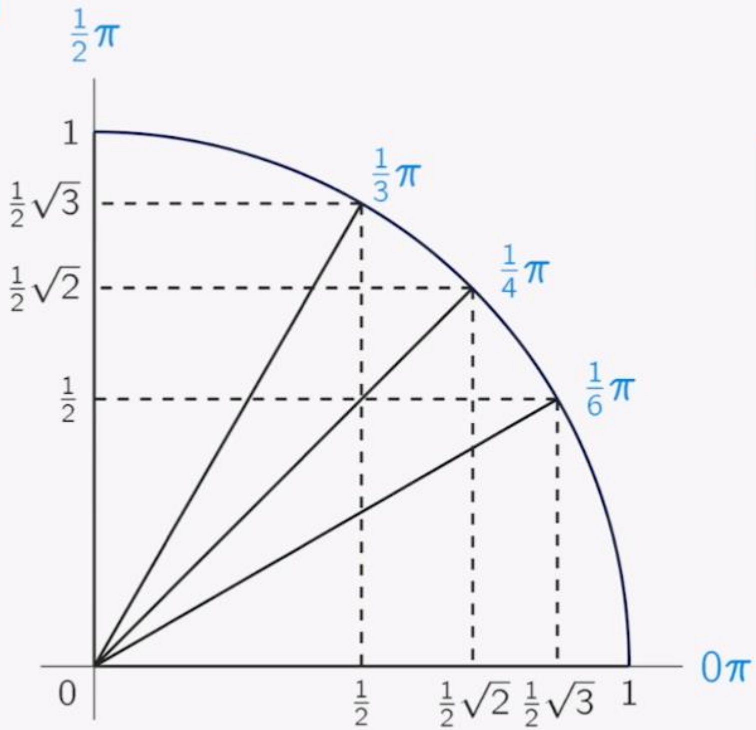




Trigonometric functions – the unit circle

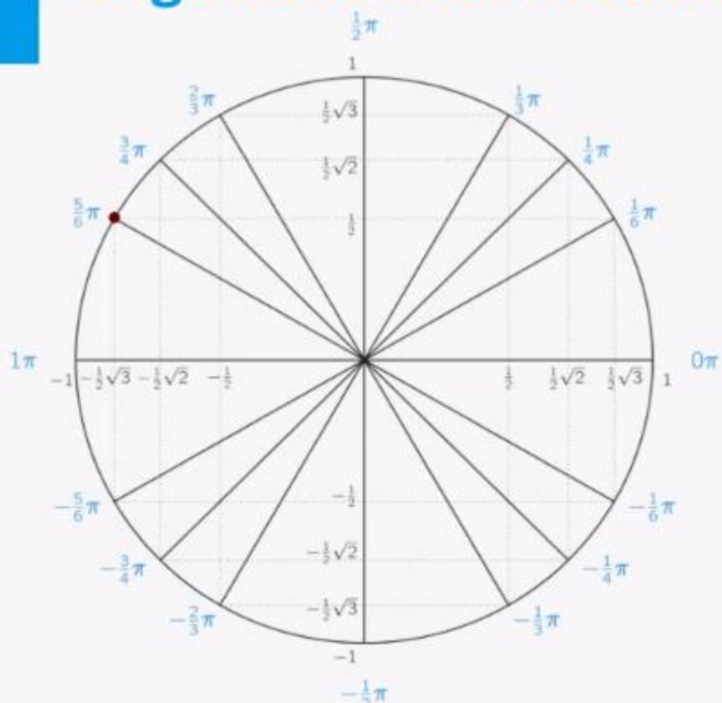


Trigonometric functions – the unit circle



	0°	30°	45°	60°	90°
θ	0	$\frac{1}{6}\pi$	$\frac{1}{4}\pi$	$\frac{1}{3}\pi$	$\frac{1}{2}\pi$
$\sin(\theta)$	0	$\frac{1}{2}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$	1
$\cos(\theta)$	1	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}$	0
$\tan(\theta)$	0	$\frac{1}{3}\sqrt{3}$	1	$\sqrt{3}$	–

Trigonometric functions - examples

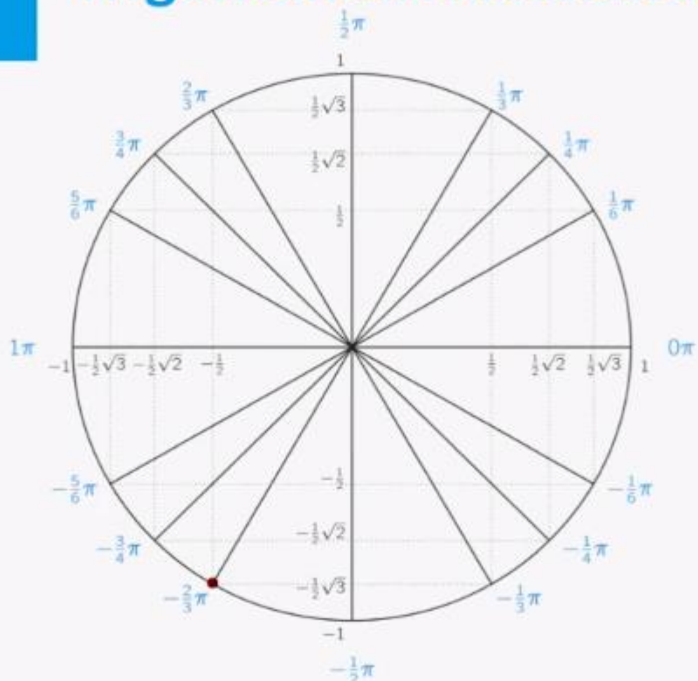


$$\cos(\tfrac{5}{6}\pi) = -\tfrac{1}{2}\sqrt{3}$$

$$\sin(\tfrac{5}{6}\pi) = \tfrac{1}{2}$$

$$\tan(\tfrac{5}{6}\pi) = \frac{\sin(\frac{5}{6}\pi)}{\cos(\frac{5}{6}\pi)} = -\tfrac{1}{3}\sqrt{3}$$

Trigonometric functions - examples



$$\cos(-\tfrac{2}{3}\pi) = -\tfrac{1}{2}$$

$$\sin(-\tfrac{2}{3}\pi) = -\tfrac{1}{2}\sqrt{3}$$

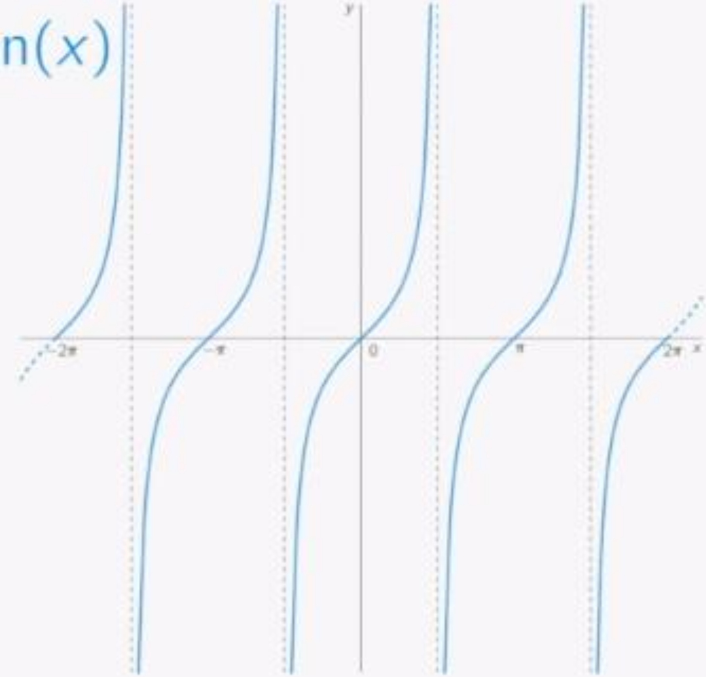
$$\tan(-\tfrac{2}{3}\pi) = \frac{\sin(-\frac{2}{3}\pi)}{\cos(-\frac{2}{3}\pi)} = \sqrt{3}$$

Trigonometric functions - graphs

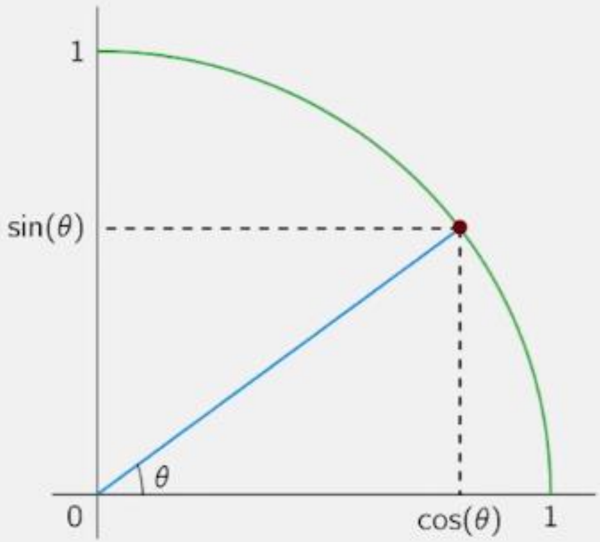
cos(x)

sin(x)

tan(x)



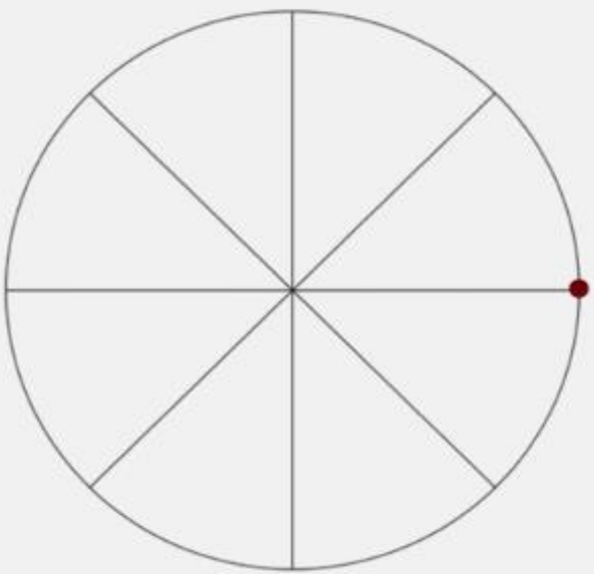
Pythagorean Theorem



Pythagorean Theorem:

$$\cos^2(\theta) + \sin^2(\theta) = 1$$

Periodicity

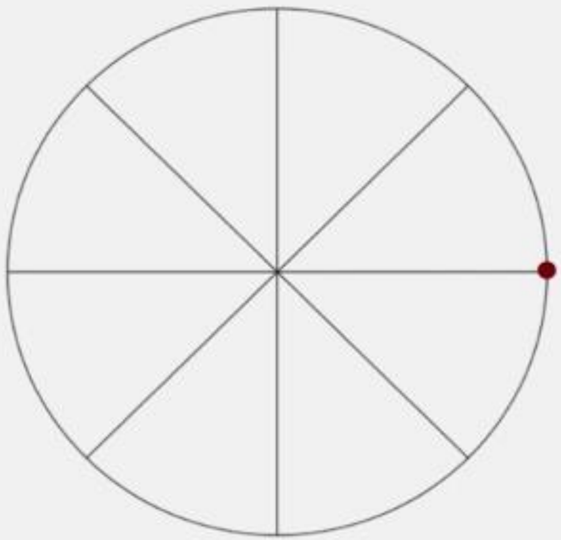


Periodicity:

$$\cos(\theta + 2\pi) = \cos(\theta)$$

$$\sin(\theta + 2\pi) = \sin(\theta)$$

Periodicity



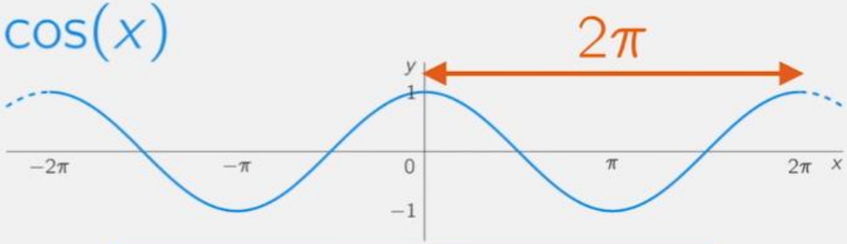
Periodicity:

$$\cos(\theta \pm 2\pi) = \cos(\theta)$$

$$\sin(\theta \pm 2\pi) = \sin(\theta)$$

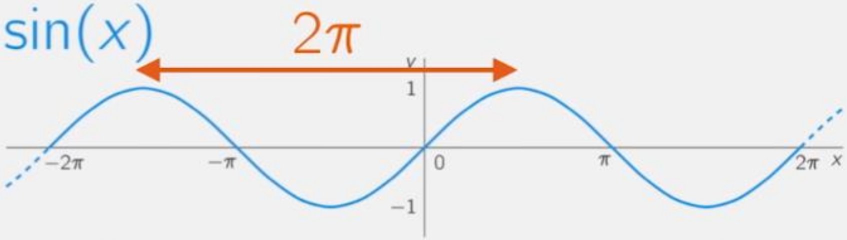
Periodicity - graphs

cos(x)



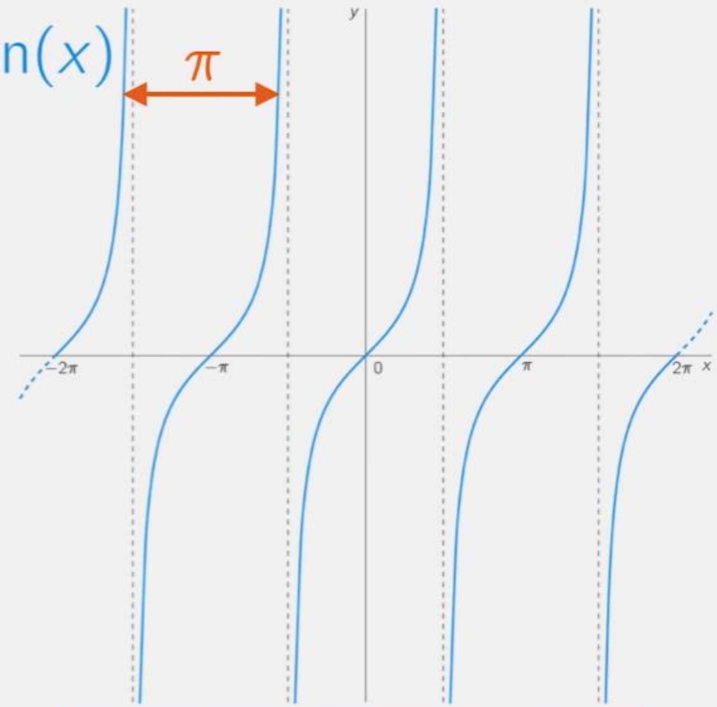
$$\cos(x \pm 2\pi) = \cos(x)$$

sin(x)



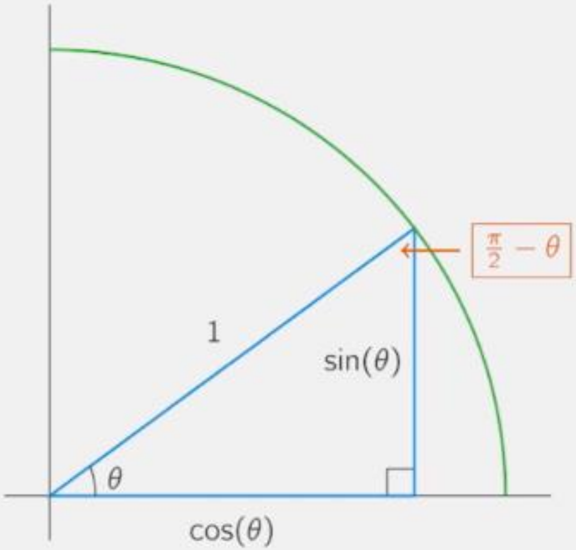
$$\sin(x \pm 2\pi) = \sin(x)$$

tan(x)



$$\tan(x \pm \pi) = \tan(x)$$

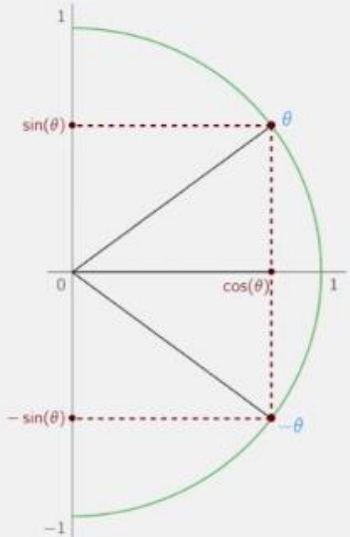
More rules of calculation



$$\cos(\tfrac{\pi}{2} - \theta) = \sin(\theta)$$

$$\sin(\tfrac{\pi}{2} - \theta) = \cos(\theta)$$

More rules of calculation



$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin(\theta)$$

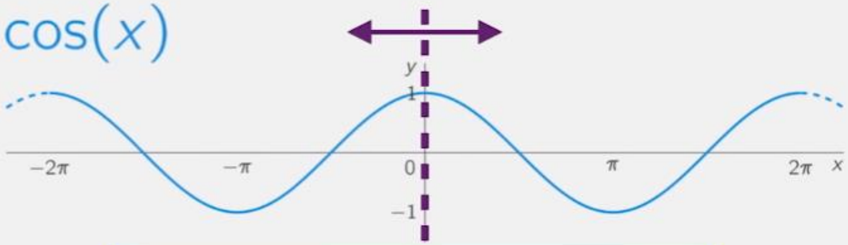
$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos(\theta)$$

$$\cos(-\theta) = \cos(\theta)$$

$$\sin(-\theta) = -\sin(\theta)$$

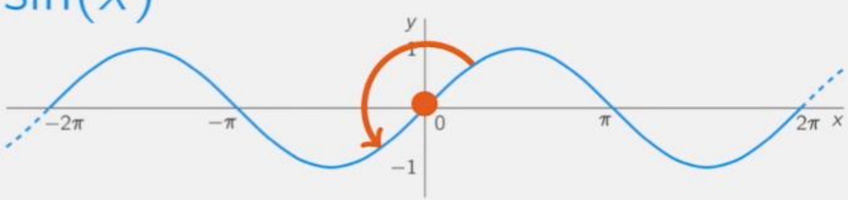
More rules of calculation - graphs

$\cos(x)$



$$\cos(-x) = \cos(x)$$

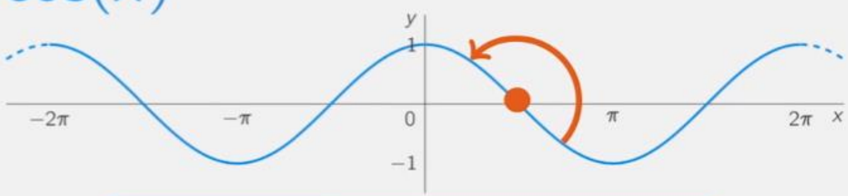
$\sin(x)$



$$\sin(-x) = -\sin(x)$$

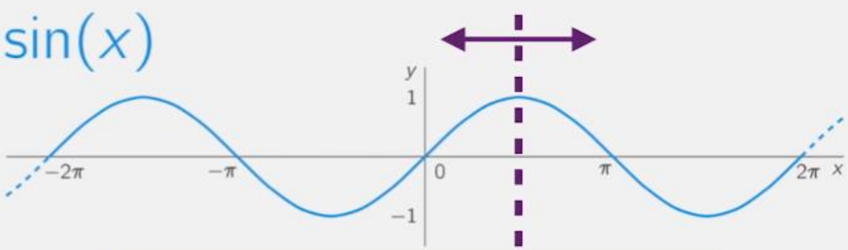
More rules of calculation - graphs

$\cos(x)$



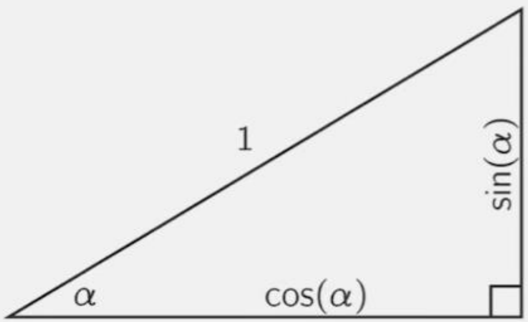
$$\cos(\pi - x) = -\cos(x)$$

$\sin(x)$

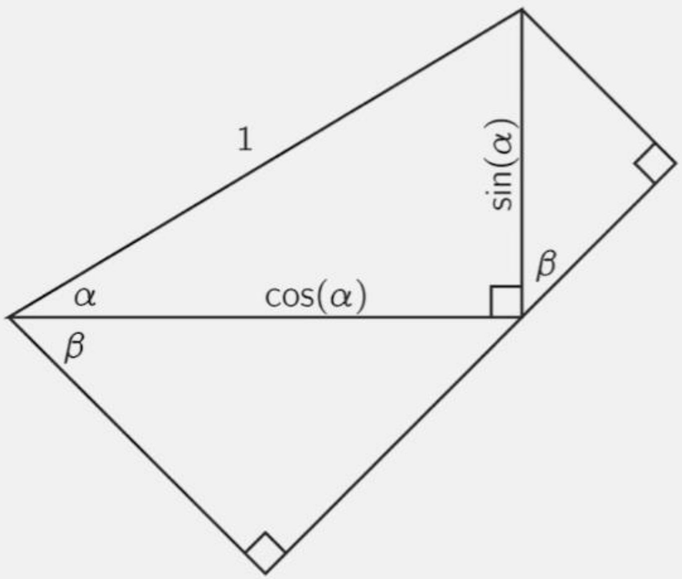


$$\sin(\pi - x) = \sin(x)$$

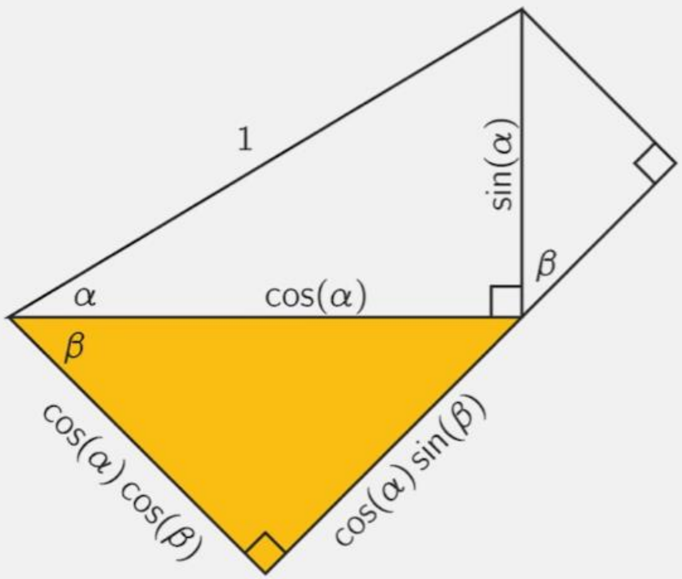
$\sin(\alpha + \beta)$ **and** $\cos(\alpha + \beta)$



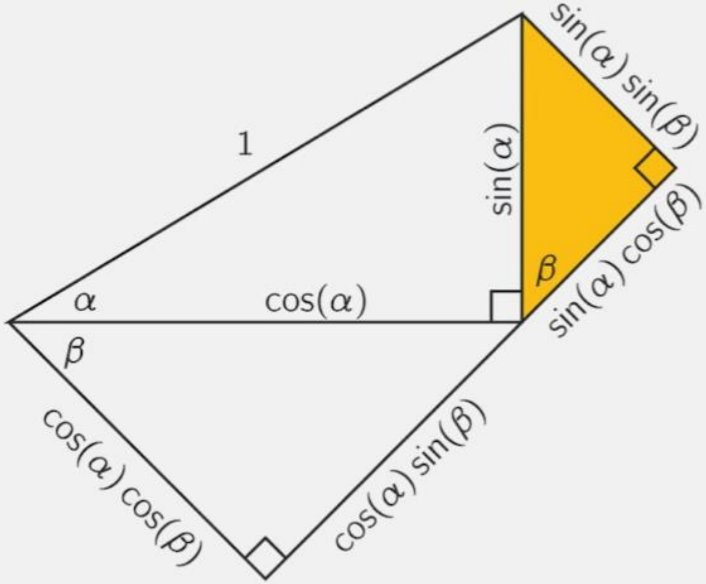
$\sin(\alpha + \beta)$ **and** $\cos(\alpha + \beta)$



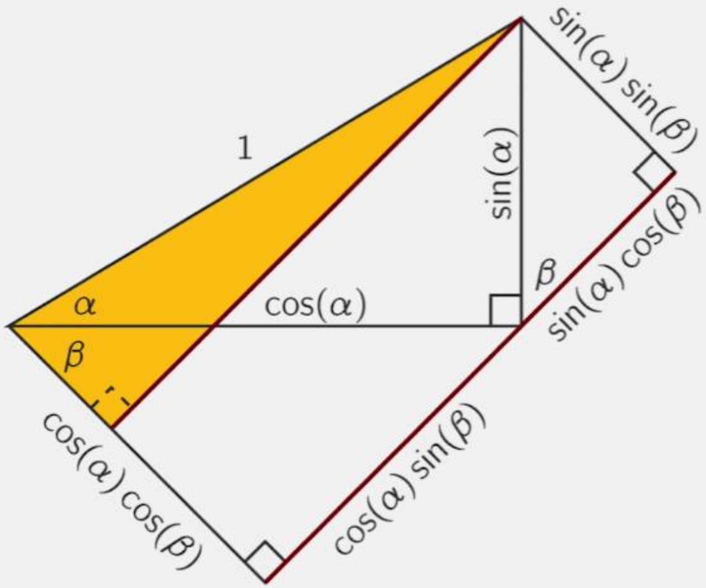
$\sin(\alpha+\beta)$ **and** $\cos(\alpha+\beta)$



$\sin(\alpha + \beta)$ **and** $\cos(\alpha + \beta)$

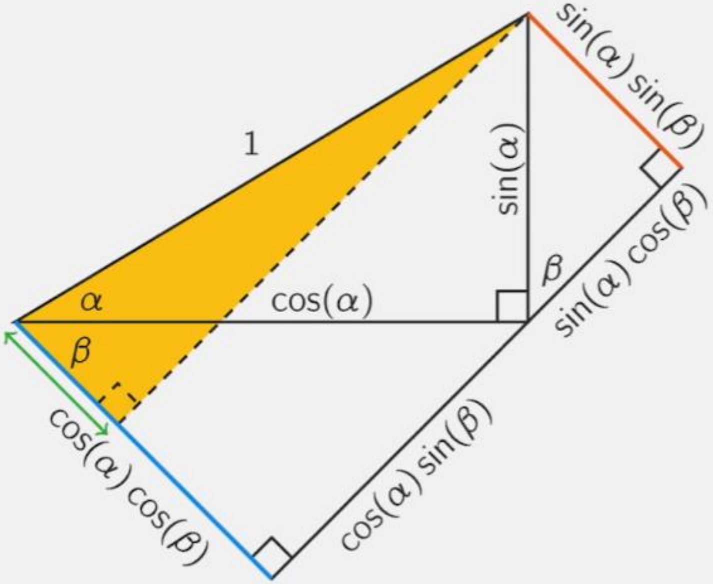


$\sin(\alpha+\beta)$ and $\cos(\alpha+\beta)$



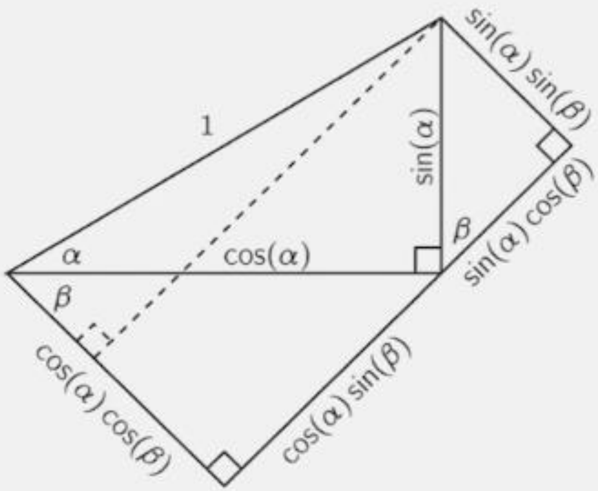
$$\sin(\alpha+\beta) = \cos(\alpha)\sin(\beta) + \sin(\alpha)\cos(\beta)$$

$\sin(\alpha + \beta)$ **and** $\cos(\alpha + \beta)$



$$\sin(\alpha + \beta) =$$
$$\cos(\alpha)\sin(\beta) + \sin(\alpha)\cos(\beta)$$
$$\cos(\alpha + \beta) =$$
$$\cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

Double angle formulas



$$\sin(\alpha + \beta) = \cos(\alpha)\sin(\beta) + \sin(\alpha)\cos(\beta)$$

$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

Compositions

outer function

A function $f(g(x))$ is the composition of f and g.

inner function

$$\sin(x^2)$$

An example

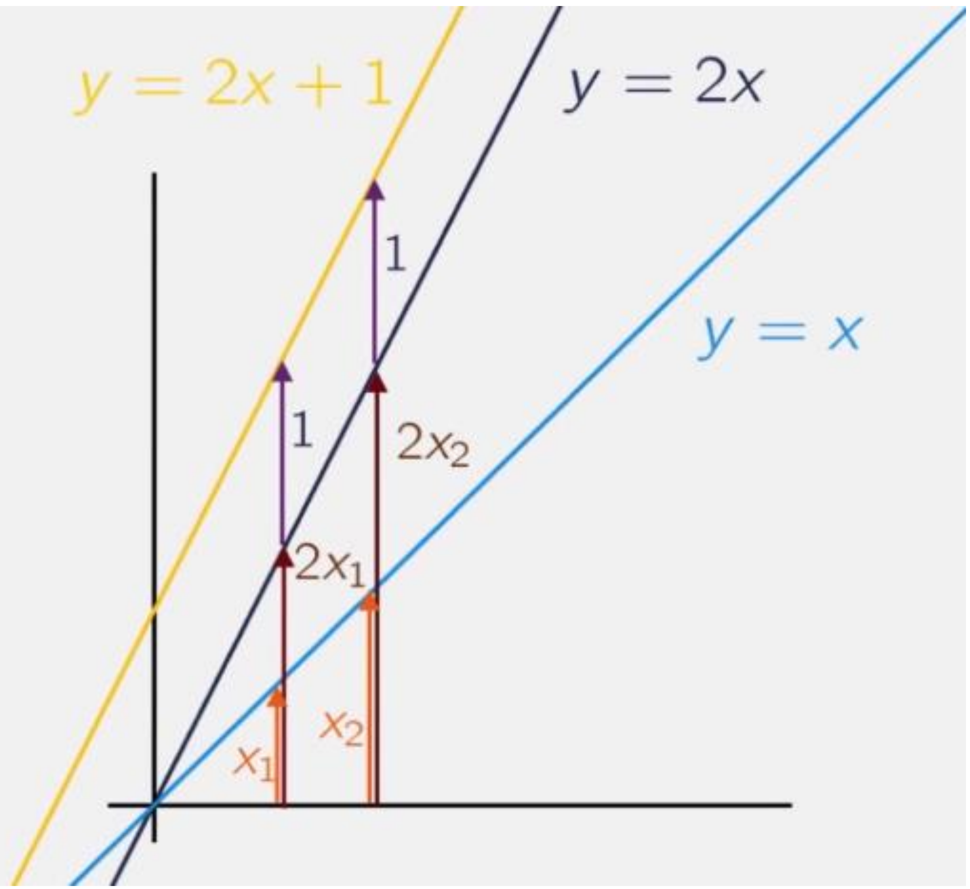
$$(x^4)^2 + 2x^4 + 1 = 4$$
$$p^2 + 2p + 1 = 4$$
$$p^2 + 2p - 3 = 0$$
$$(p+3)(p-1) = 0$$
$$p = -3 \quad \text{or} \quad p = 1$$
$$x^4 = -3 \quad \text{or} \quad x^4 = 1$$
✖ $\quad x = 1$ or $x = -1$

$$f(x) = x^2 + 2x + 1$$
$$g(x) = x^4$$

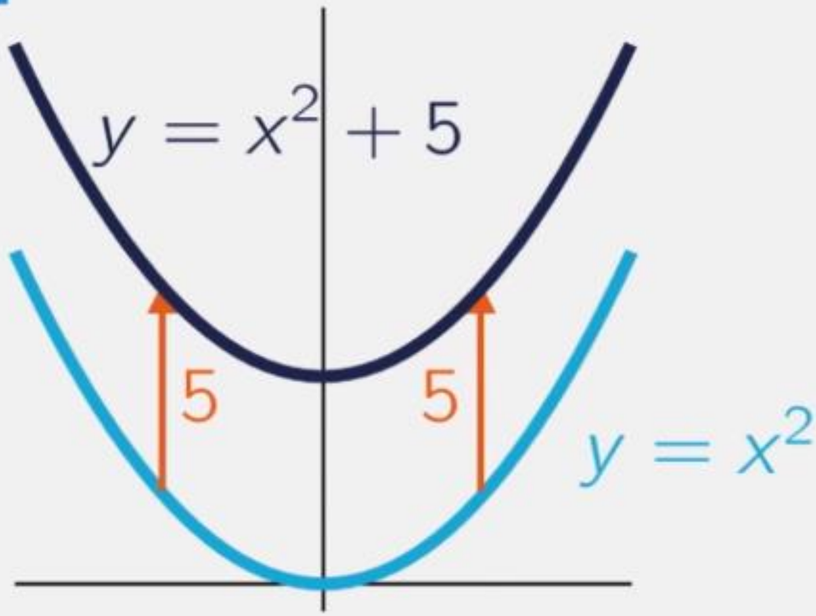
Linear Functions

$$f(x) = 2x + 1$$

- Scaling $\times 2$
- Translation $+1$

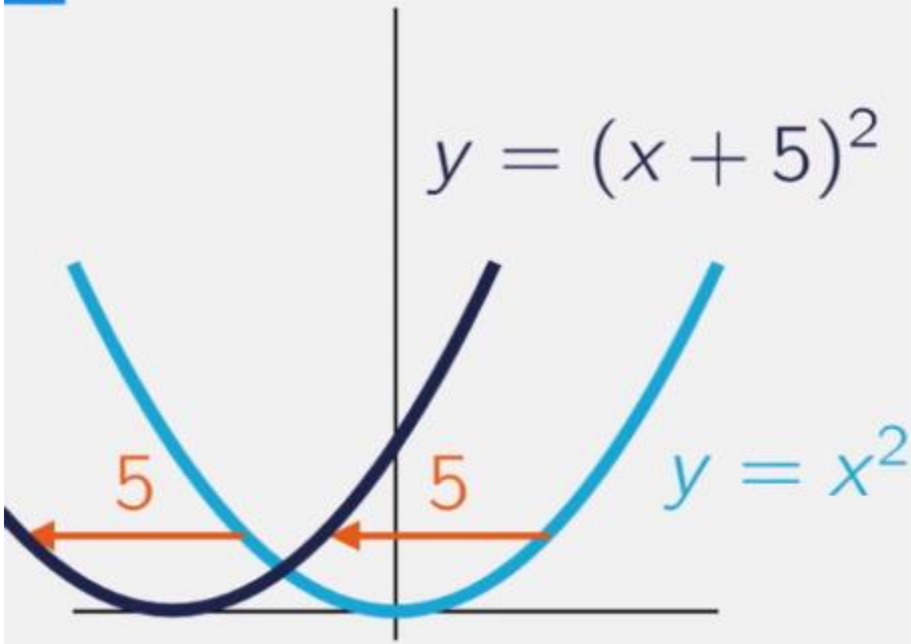


Vertical Translation: The graph of $g(x) + a$



$$f(x) = x + 5$$
$$g(x) = x^2$$
$$f(g(x)) = x^2 + 5$$

Horizontal Translation: The graph of $g(x+a)$

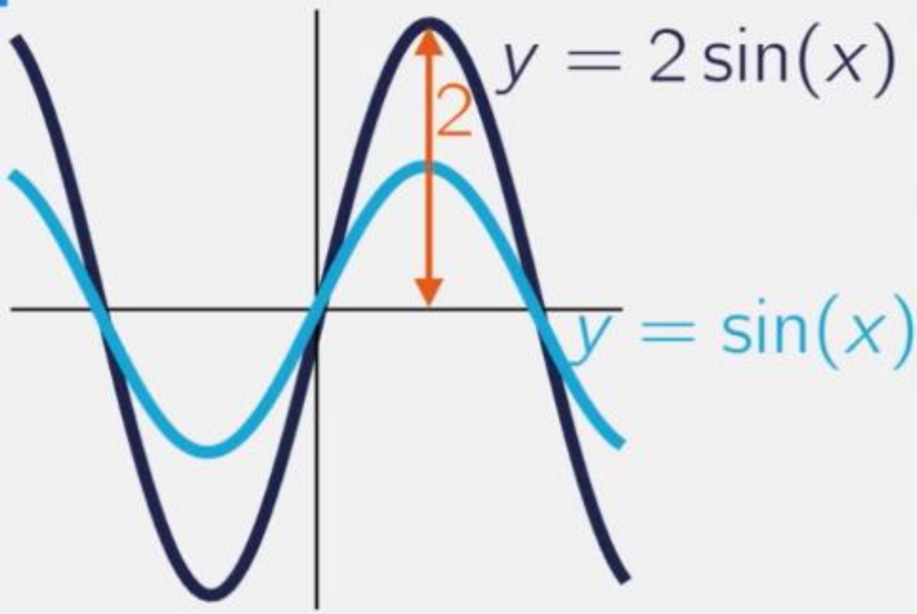


$$f(x) = x + 5$$

$$g(x) = x^2$$

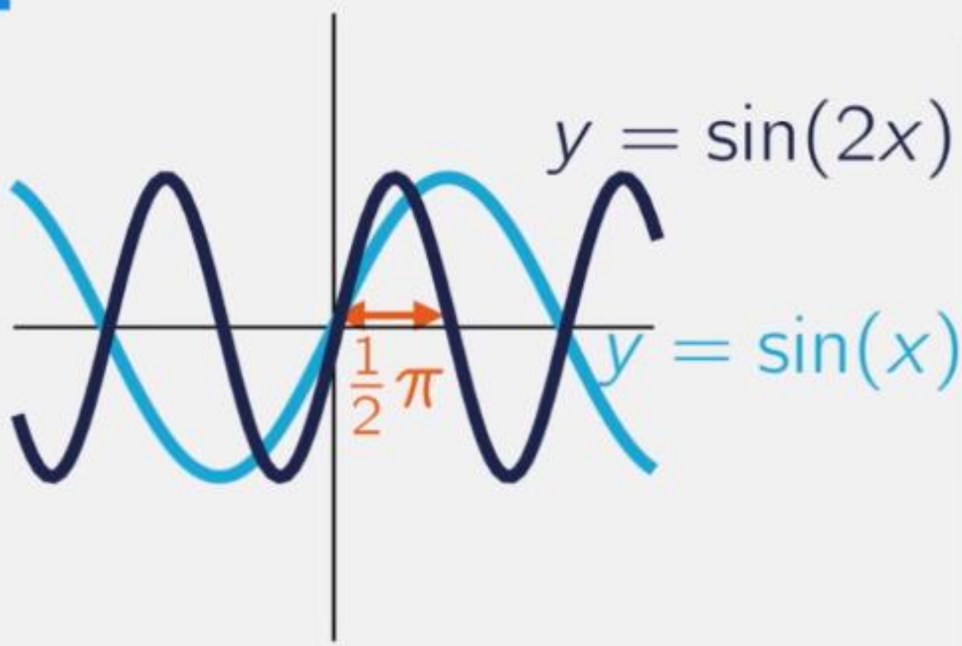
$$g(f(x)) = (x+5)^2$$

Vertical Scaling: The graph of $ag(x)$



$$f(x) = 2x$$
$$g(x) = \sin(x)$$
$$f(g(x)) = 2\sin(x)$$

Vertical Scaling: The graph of $g(ax)$



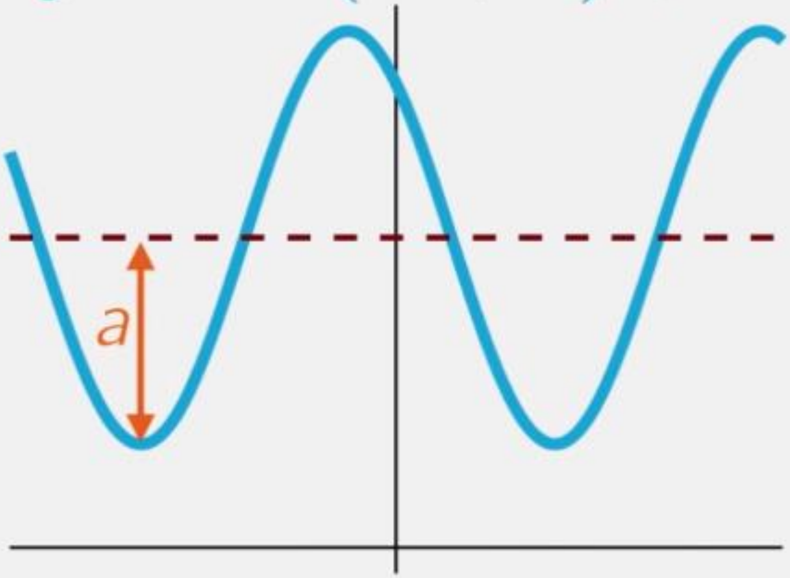
$$f(x) = 2x$$
$$g(x) = \sin(x)$$
$$g(f(x)) = \sin(2x)$$

Composing with linear functions

$g(x) + a$	**Vertical shift upwards**
$g(x + a)$	**Horizontal shift to the left**
$ag(x)$	**Vertical scaling by** a
$g(ax)$	**Horizontal scaling by** $1/a$

A wave function

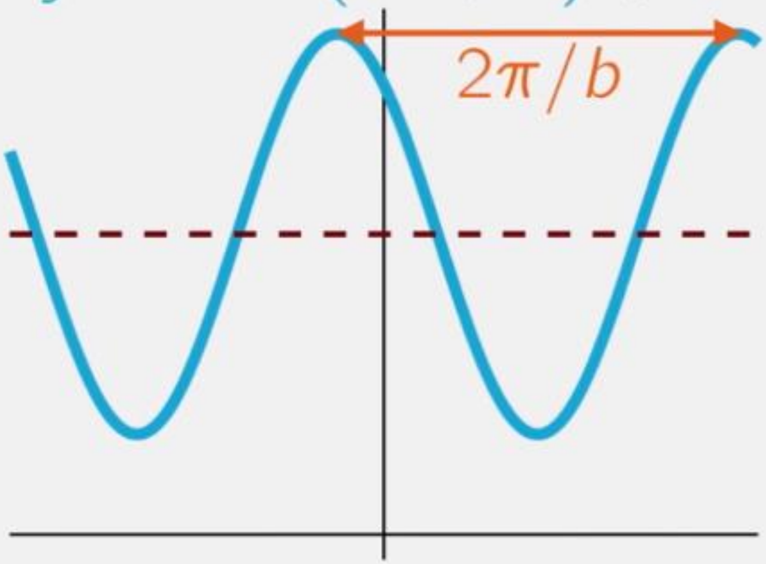
$$y = a\sin(bt + c) + d$$



a	**Amplitude**

A wave function

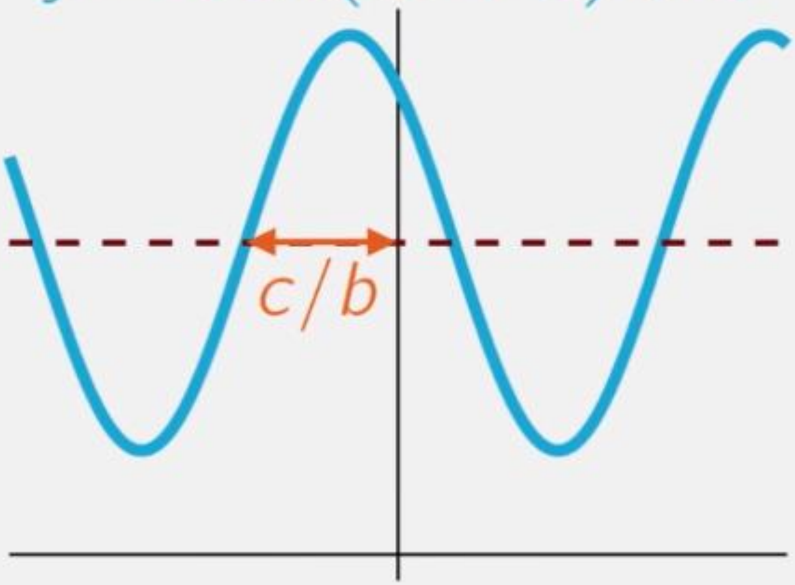
$$y = a\sin(bt + c) + d$$



a	**Amplitude**
$2\pi/b$	**Period**

A wave function

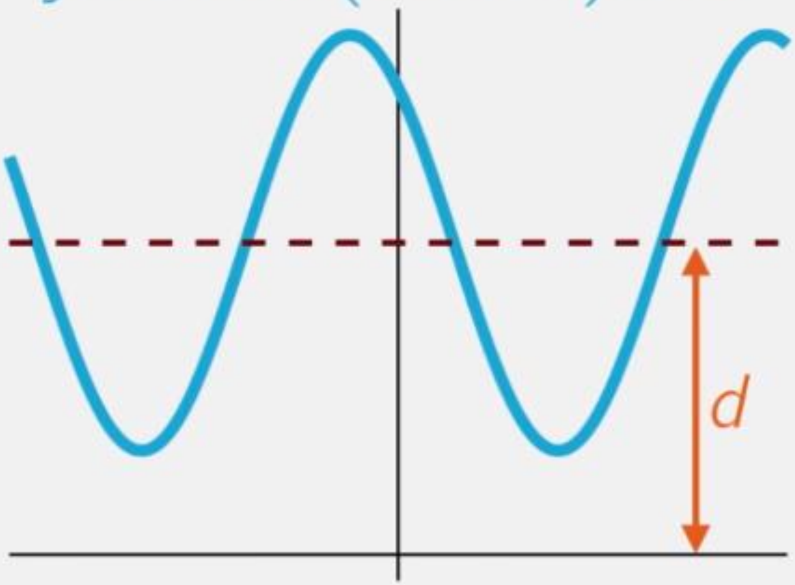
$$y = a\sin(bt + c) + d$$



a	**Amplitude**
$2\pi/b$	**Period**
c	**Phase**

A wave function

$$y = a\sin(bt + c) + d$$



a	**Amplitude**
$2\pi/b$	**Period**
c	**Phase**
d	**Equilibrium height**