

Funções Exponenciais

**DelftX: CalcSP01x Pre-University
Calculus (Self-Paced)**

2.4 Exponential functions

Exponential functions

Form of an exponential function:

$$A b^x$$

$$A \neq 0$$

$$b > 0, b \neq 1$$

Example: 200×1.03^x

$$A \quad b$$

- b : base
- x : exponent

Rules of calculation

- $x^{A+B} = x^A x^B$

- $x^{A-B} = \frac{x^A}{x^B}$

- $(x^A)^B = x^{AB}$

- $x^A y^A = (xy)^A$



- $b^{x+y} = b^x b^y$

- $b^{x-y} = \frac{b^x}{b^y}$

- $(b^x)^y = b^{xy}$

- $b^x c^x = (bc)^x$

$$b^{\frac{p}{q}} = (b^p)^{\frac{1}{q}} = \sqrt[q]{b^p}$$

What is b^x if x is not rational?

Approximation of x with rational numbers



Approximation of b^x

$$2^{\sqrt{2}} = ?$$

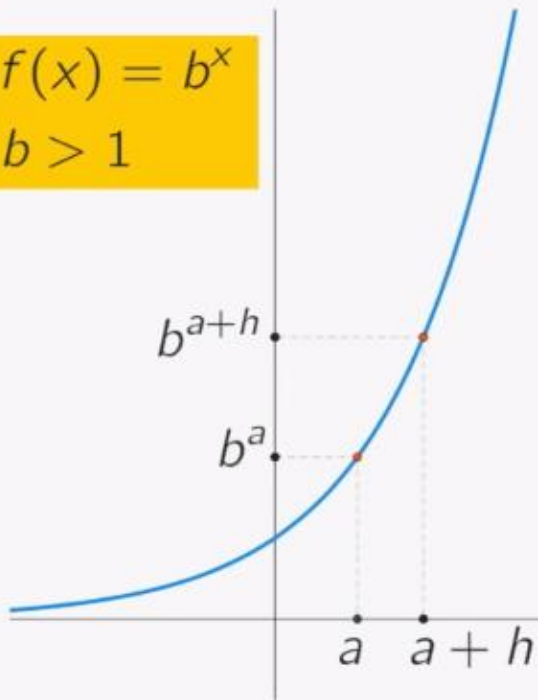
1.4
1.41
1.414
1.4142
⋮
 $\sqrt{2}$

$$\begin{aligned} 2^{1.4} &= 2^{\frac{14}{10}} = 2.6390\dots \\ 2^{1.41} &= 2^{\frac{141}{100}} = 2.6574\dots \\ 2^{1.414} &= 2^{\frac{1414}{1000}} = 2.6647\dots \\ 2^{1.4142} &= 2^{\frac{14142}{10000}} = 2.6651\dots \\ &\vdots \end{aligned}$$

$$2^{\sqrt{2}} = 2.6651\dots$$

Graph of b^x

$$f(x) = b^x$$
$$b > 1$$



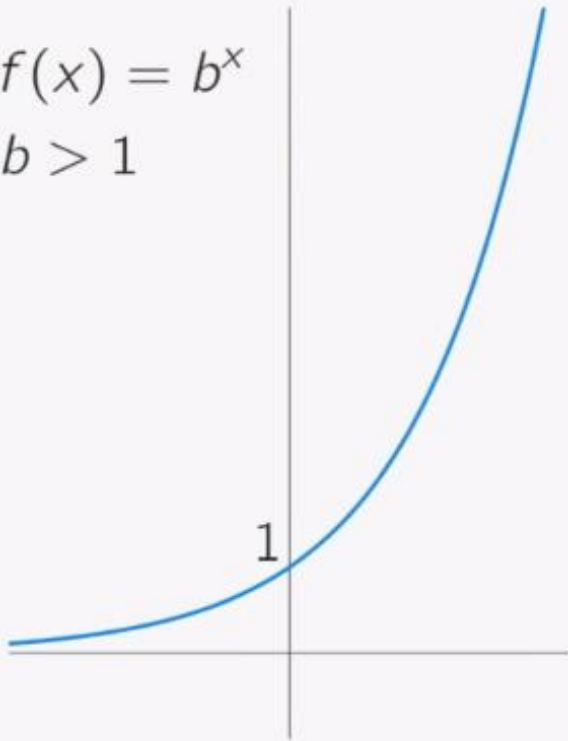
$$b^{a+h} = b^a b^h > b^a$$

$f(a+h) > f(a)$ → increasing

Graph of b^x

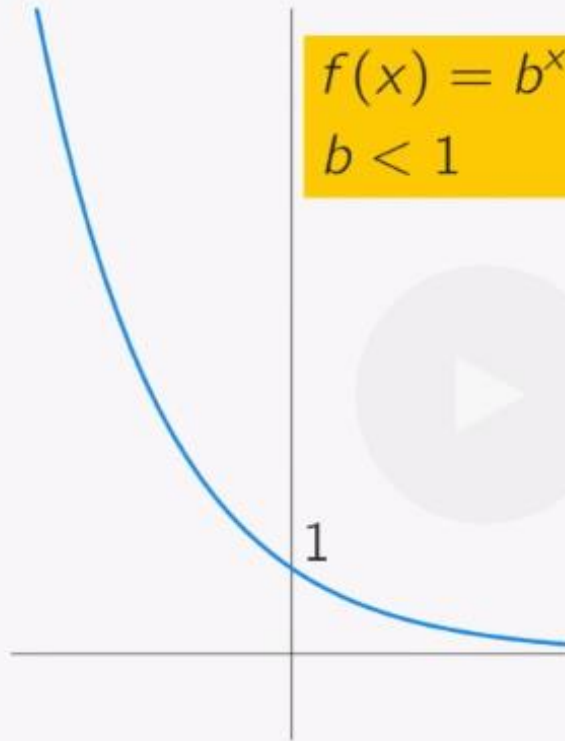
$$f(x) = b^x$$

$$b > 1$$

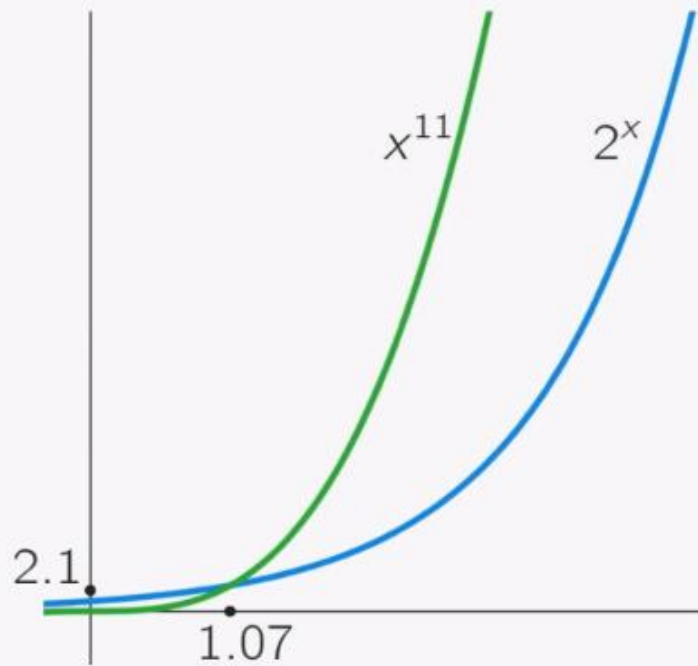


$$f(x) = b^x$$

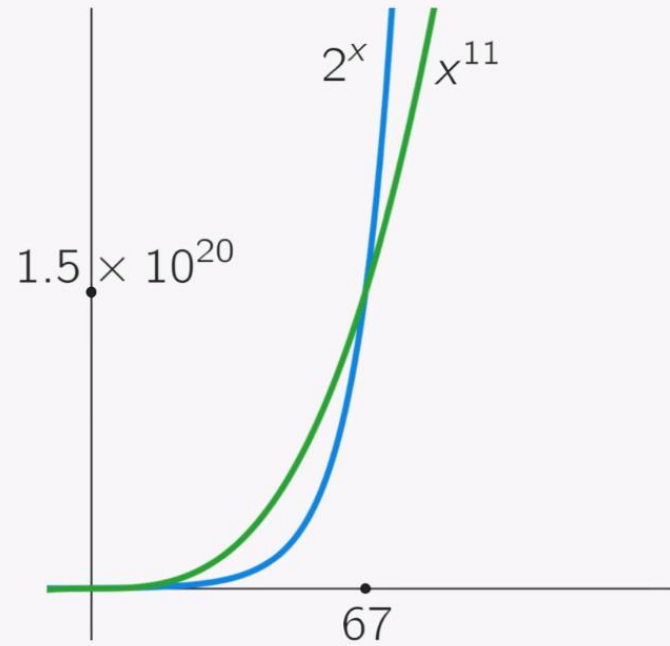
$$b < 1$$



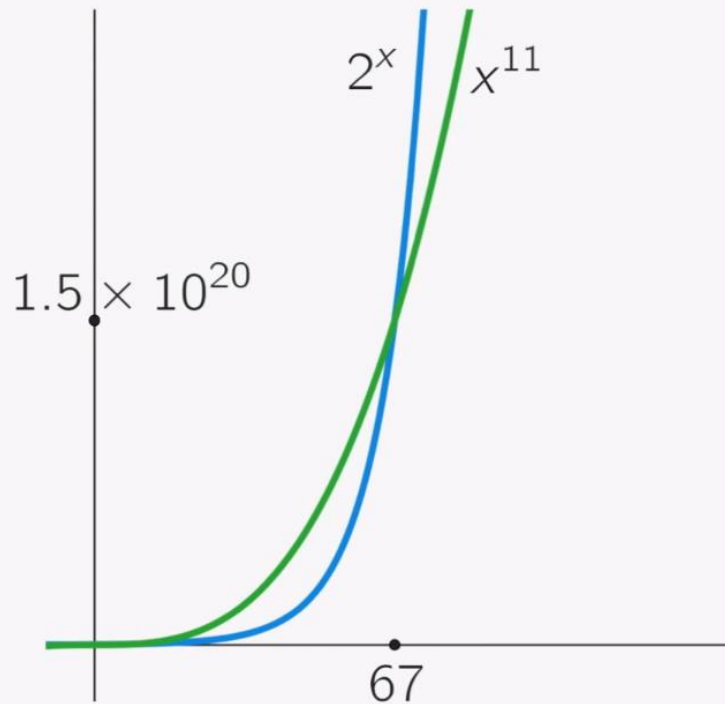
Comparison with x^n



Comparison with x^n



Comparison with x^n



b^x with $b > 1$ grows faster than x^n for any number n

Base e

- Number e is often used as base
- $e = 2.7182818284590452353602874713527 \dots$
- e is the **unique** number such that

$$\frac{d}{dx} e^x = e^x$$

Summary

- Exponential function: $A b^x$
- Rules of calculation
- b^x , $b > 1$, grows faster than x^n

- $b^{x+y} = b^x b^y$
- $b^{x-y} = \frac{b^x}{b^y}$
- $(b^x)^y = b^{xy}$
- $b^x c^x = (bc)^x$