Funções Potência

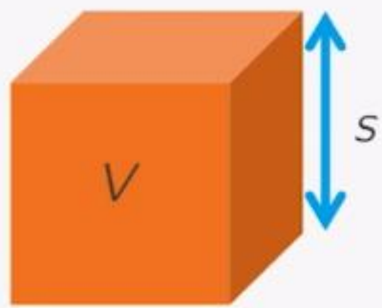
DelftX: CalcSP01x Pre-University Calculus (Self-Paced)

1.5 Power functions

Power functions

$f(x) = x^a$, where a is a constant

$A = \pi r^2$



$s = V^{\frac{1}{3}}$

$F \propto r^{-2}$

Power functions with integer exponents

$f(x) = x^a$ for integer $a \geq 0$

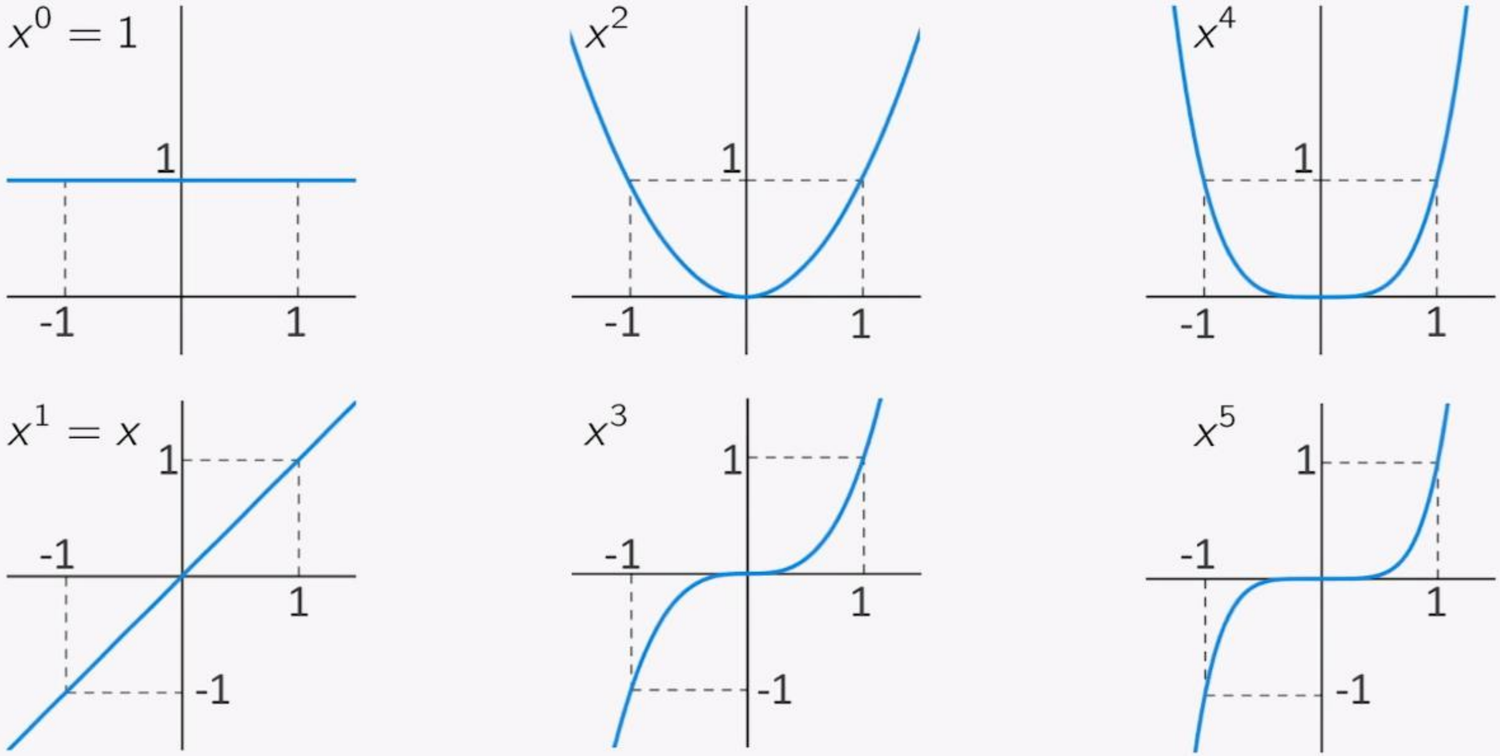
Rules of calculation

$$x^a x^b = \underbrace{x \cdot x \cdots x}_{a} \cdot \underbrace{x \cdot x \cdots x}_{b} = \underbrace{x \cdot x \cdots x}_{a+b} = x^{a+b}$$

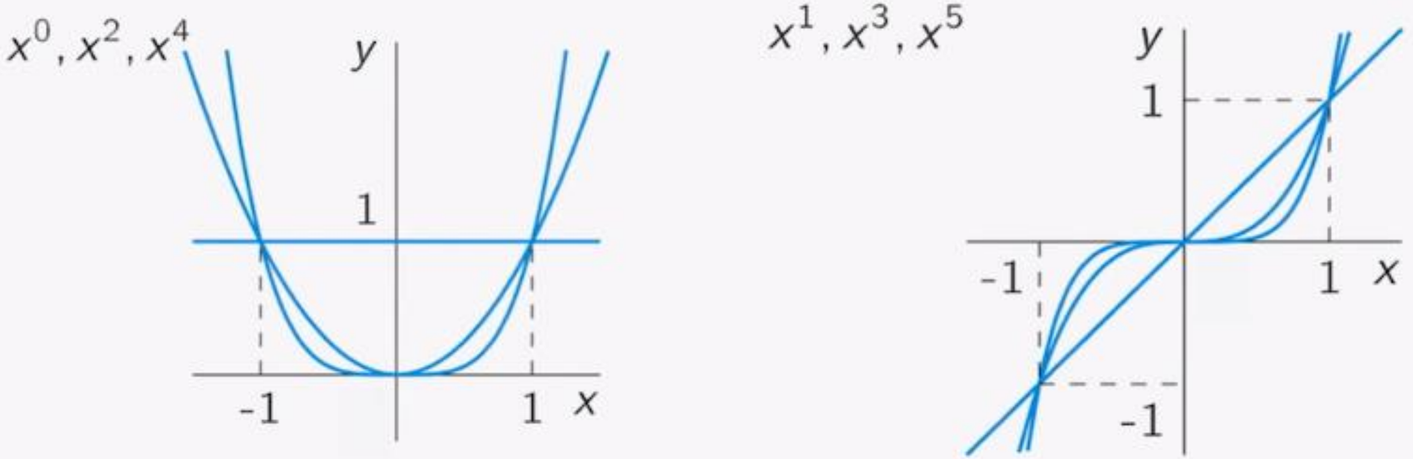
$$(x^a)^b = \underbrace{x^a \cdot x^a \cdots x^a}_{b} = x^{a \cdot b}$$

$$(x\,y)^a = \underbrace{xy \cdot xy \cdots xy}_{a} = \underbrace{x \cdot x \cdots x}_{a} \cdot \underbrace{y \cdot y \cdots y}_{a} = x^a\,y^a$$

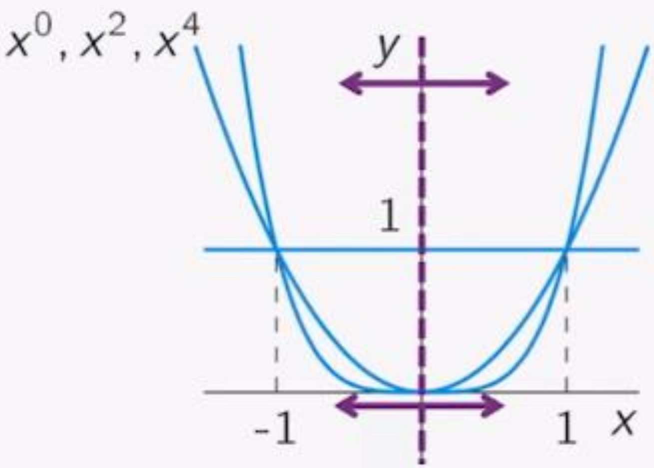
Graphs



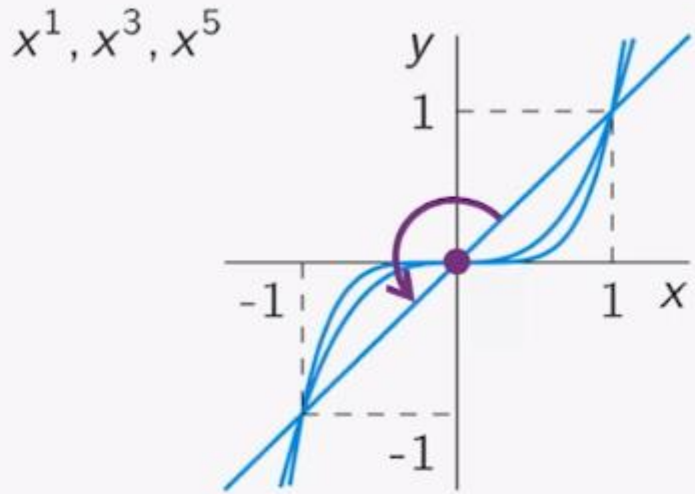
Graphs



Graphs



even: $f(-x) = f(x)$



odd: $f(-x) = -f(x)$

Power functions: negative exponent

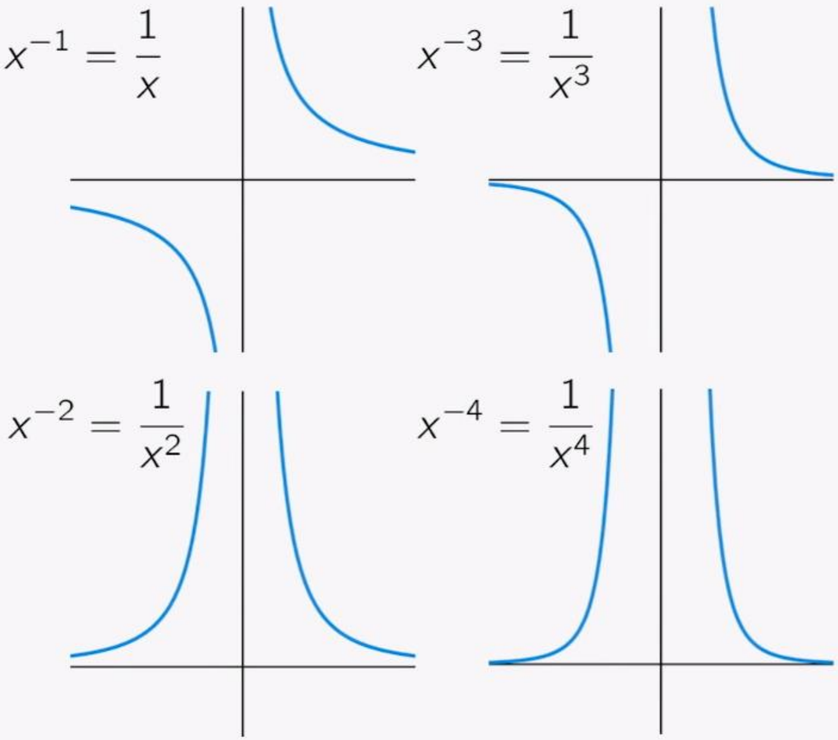
$f(x) = x^a$ for *negative* integer a

Example: What is x^{-3}? Rules of calculation: $x^3 \cdot x^{-3} = x^0 = 1$

This implies that: $x^{-3} = \frac{1}{x^3}$

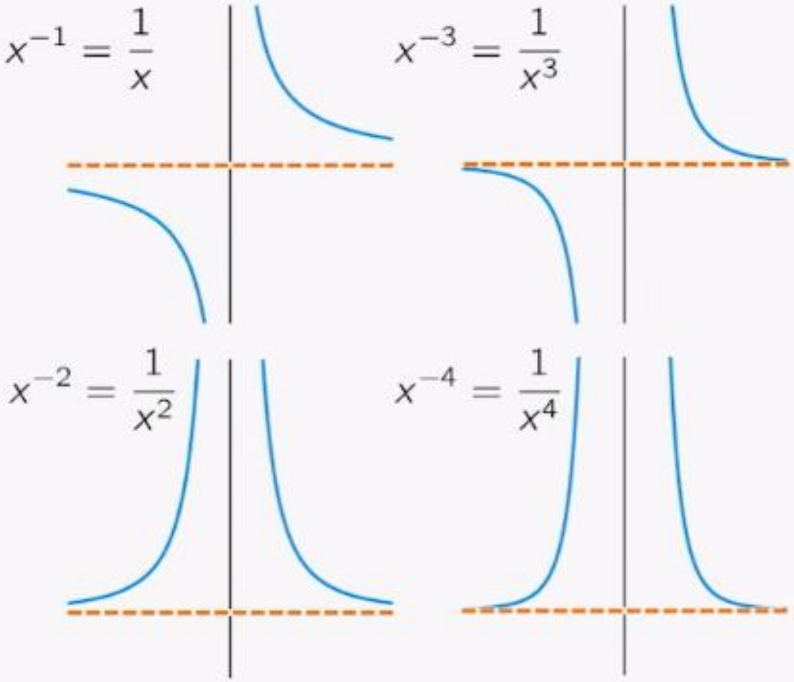
$$x^{-1} = \frac{1}{x}, \; x^{-2} = \frac{1}{x^2}, \; x^{-3} = \frac{1}{x^3},$$

Graphs



$f(x) = x^a$, a negative

Graphs

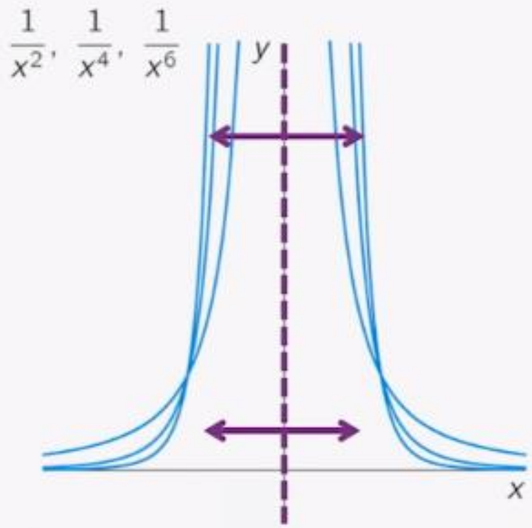


$f(x) = x^a$, a negative

Graph has

- vertical asymptote at $x = 0$
- horizontal asymptote at $y = 0$

Graphs



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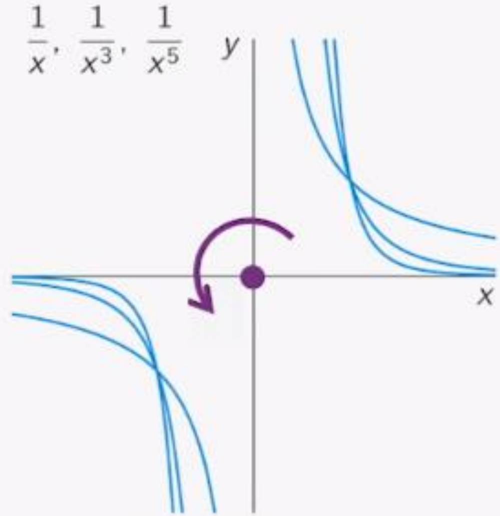


odd: $f(-x) = -f(x)$

Rules of calculation

$f(x) = x^a$, a constant

Rules of calculation

$$x^a x^b = x^{a+b}$$
$$\frac{x^a}{x^b} = x^{a-b}$$
$$(x^a)^b = x^{a \cdot b}$$
$$(x\,y)^a = x^a\,y^a$$

Other power functions

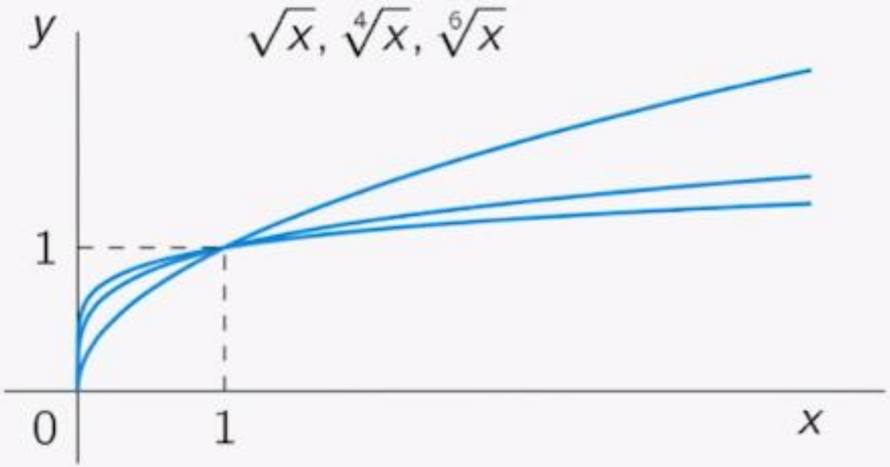
$f(x) = x^a \quad (x > 0)$

Example: What is $x^{\frac{1}{4}}$? Rules of calculation: $\left(x^{\frac{1}{4}}\right)^4 = x^1 = x$

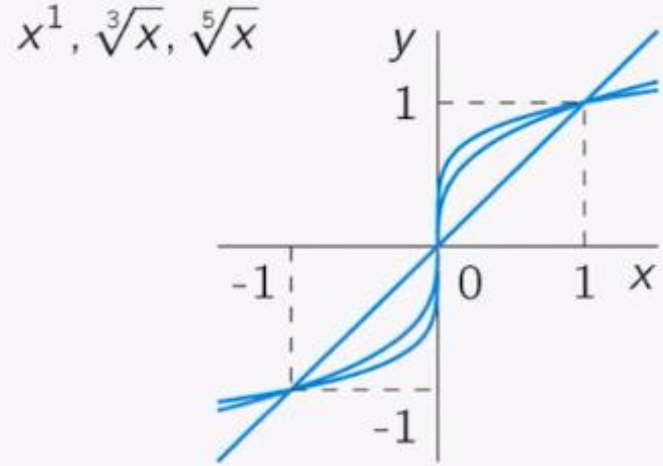
This implies that: $x^{\frac{1}{4}} = \sqrt[4]{x}$

$a = \frac{1}{n}$ with $n = 1, 2, 3, \ldots$: $x^{\frac{1}{n}} = \sqrt[n]{x}$

Graphs

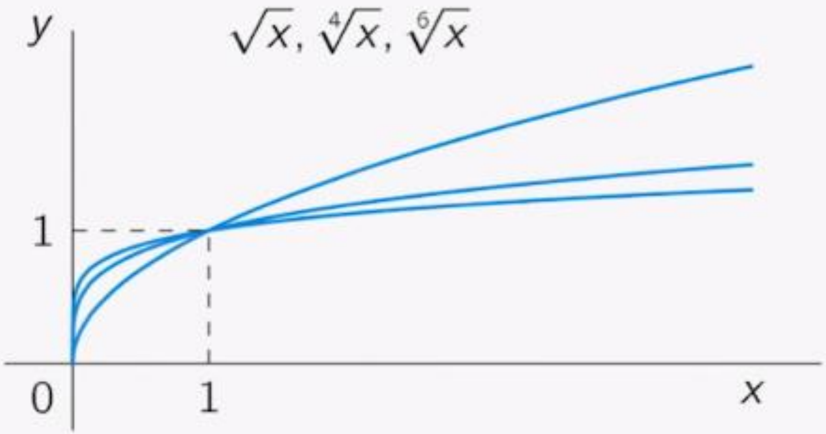


domain: $[0, \infty)$

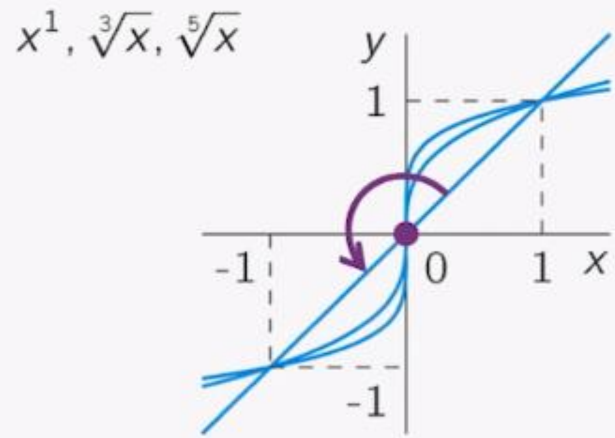


domain: $(-\infty, \infty)$

Graphs



domain: $[0, \infty)$



odd: $f(-x) = -f(x)$

More power functions

Examples:

- $x^{\frac{2}{3}} = (x^2)^{\frac{1}{3}} = \sqrt[3]{x^2}$

  $x^{\frac{2}{3}} = (x^{\frac{1}{3}})^2 = (\sqrt[3]{x})^2$

- $x^{-\frac{5}{2}} = (\frac{1}{x^5})^{\frac{1}{2}} = \frac{1}{\sqrt{x^5}}$

  $x^{-\frac{5}{2}} = (\sqrt{x})^{-5} = \frac{1}{(\sqrt{x})^5}$

$$x^{\frac{p}{q}} = \sqrt[q]{x^p} = (\sqrt[q]{x})^p$$

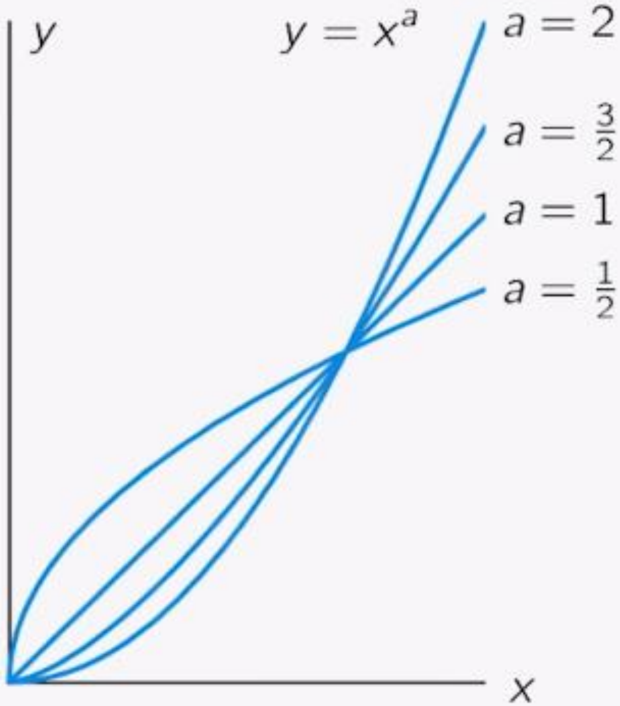
$$x^{-\frac{p}{q}} = \frac{1}{\sqrt[q]{x^p}} = \frac{1}{(\sqrt[q]{x})^p}$$

General properties

For which values of x is x^a defined?

	$a \geq 0$	$a < 0$
In general	$x \geq 0$	$x > 0$
Integer a ... or $a = \frac{p}{q}$ with q odd	all x	$x \neq 0$

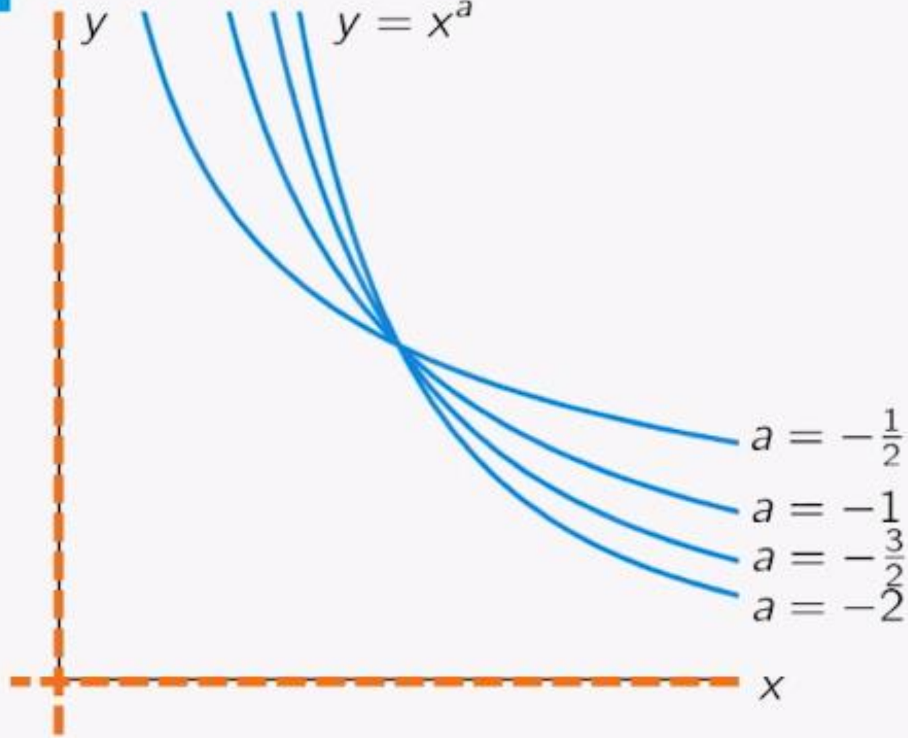
General properties



Graphs $(x > 0)$

- $a > 0$: increasing

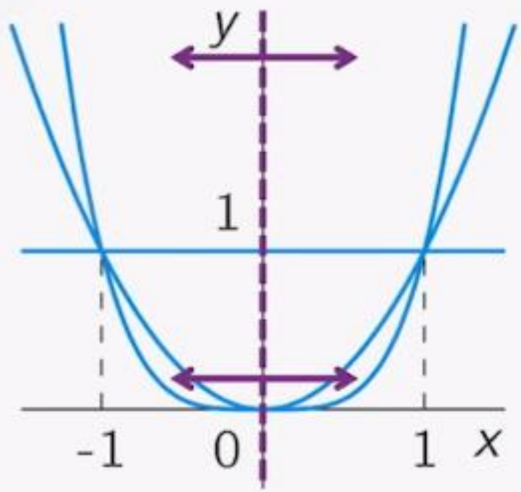
General properties



Graphs

- $a > 0$: increasing for $x > 0$
- $a < 0$: decreasing for $x > 0$ asymptotes at $x = 0$ and $y = 0$

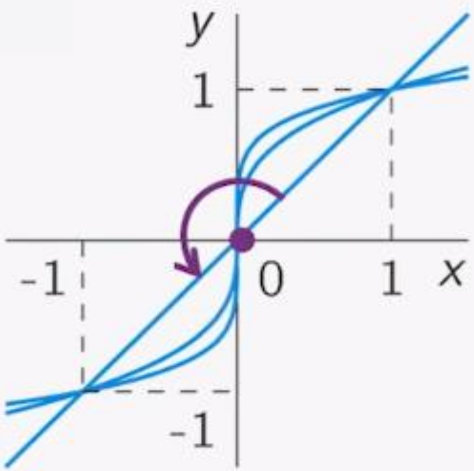
General properties



If defined for $x < 0$:

- either *line symmetry* in *y*-axis

General properties



If defined for $x < 0$:

- either *line symmetry* in *y*-axis
- or *point symmetry* in origin

Power functions

Rules of calculation

$$x^a x^b = x^{a+b}$$
$$\frac{x^a}{x^b} = x^{a-b}$$
$$(x^a)^b = x^{a \cdot b}$$
$$(x\,y)^a = x^a\,y^a$$

For $x, y > 0$: always true

For other x or y: depends on a and b!