NOTES AND CORRESPONDENCE

The Effect of Neglecting the Virtual Temperature Correction on CAPE Calculations

CHARLES A. DOSWELL III AND ERIK N. RASMUSSEN*

NOAA/Environmental Research Laboratories, National Severe Storms Laboratory, Norman, Oklahoma

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ABSTRACT

A simple theoretical analysis of the impact of neglecting the virtual correction on calculation of CAPE is made. This theory suggests that while ignoring the virtual correction does not introduce much error for large CAPE values, the relative error can become substantial for small CAPE. A test of the theory is done by finding the error made by ignoring the virtual correction to CAPE for all the soundings in 1992 having positive CAPE (when the correction is made). Results of this empirical test confirm that the relative error made in ignoring the correction increases with decreasing CAPE. A number of other "corrections" to CAPE might be considered. In a discussion of the issues associated with the results of the analysis, it is recommended that CAPE calculations should include the virtual correction but that other complications should be avoided for most purposes, especially when making comparisons of CAPE values. A standardized CAPE calculation also is recommended.

1. Introduction

This note addresses the calculation of convective available potential energy (CAPE). CAPE is a quantity most closely associated with the environment in which deep convection might occur, and has become widely accepted as a *forecasting* parameter with the advent of computer programs that calculate CAPE from operational soundings or model forecasts. Recently, it has come to our attention that the algorithms for computing CAPE are not all the same. In particular, some schemes do not include the virtual temperature correction in the calculations.

It is well known that the *virtual* temperature T_v is the proper temperature to use in the equation of state $p = \rho R T_v$ in order that the gas constant R be truly constant. Otherwise, the addition of moisture changes the "constant," with the change depending on the amount of moisture. Use of the virtual correction to temperature T such that

$$T_v = T(1 + \epsilon q), \tag{1}$$

where $\epsilon = 0.608$ when the mixing ratio q is expressed in g g^{-1}, allows the use of the gas constant for *dry* air, $R = 2.87 \times 10^2$ m^2 s^{-2} K^{-1}, in the equation of state. The virtual correction is always positive because adding water vapor to a parcel makes it less dense, which can be considered equivalent to warming the parcel.

Since CAPE concerns the *difference* in density between a rising parcel and its environment, and since an accurate calculation of density requires the virtual temperature, it should be obvious that the virtual correction is necessary when estimating CAPE. To assess the impact of ignoring the virtual correction, an analysis of the contribution from this error follows in section 2. In section 3, the application of CAPE estimates is discussed and some recommendations are made.

2. Theoretical analysis of the error

We begin with logarithmic differencing of the equation of state,

$$\frac{\delta\rho}{\rho} = \frac{\delta p}{p} - \frac{\delta T_v}{T_v}, \tag{2}$$

where the difference operator $\delta(\ )$ is between the parcel and its environment:

$$\delta(\ ) \equiv (\ )_{\text{parcel}} - (\ )_{\text{env}}.$$

The standard assumption in parcel theory is to ignore the contribution to differences in density due to differences in *pressure* between the parcel and the environment. If this standard assumption is made and (1) is logarithmically differenced, substitution into (2) gives

$$\frac{\delta\rho}{\rho} \approx -\frac{\delta T_v}{T_v},$$
$$= -\left[\frac{\delta T}{T} + \frac{\epsilon\delta q}{(1 + \epsilon q)}\right]. \tag{3}$$

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[*] Additional affiliation: Cooperative Institute for Mesoscale Meteorological Studies, Norman, Oklahoma.

Corresponding author address: Dr. Charles A. Doswell III, National Severe Storms Laboratory, 1313 Halley Circle, Norman, OK 73069.

Since $\epsilon q \ll 1$, then

$$(1 + \epsilon q)^{-1} \approx (1 - \epsilon q) + O(\epsilon^2 q^2),$$

so that (3) becomes

$$\frac{\delta \rho}{\rho} \approx -\left(\frac{\delta T}{T} + \epsilon \delta q - \epsilon^2 q \delta q\right). \qquad (4)$$

Neglecting the virtual correction means neglecting all but the first term on the right-hand side (rhs) of (4). To ascertain the error associated with this neglect, consider the following estimates of the terms, appropriate for a potentially convective situation:

(a) $\delta T \sim 1 - 10$ K
(b) $T \sim 300$ K
(c) $\delta q \sim 0.001 - 0.010$ ($1 - 10$ g kg^{-1})
(d) $q \sim 0.010$ (10 g kg^{-1}).

Therefore, the first term on the rhs of (4), $|\delta T/T| \sim (1 - 10)/300 \sim 3 \times (10^{-3} - 10^{-2})$; the second, $|(\epsilon \delta q)| \sim 6 \times (10^{-4} - 10^{-3})$; and the third, $|(\epsilon^2 q \delta q)| \sim 3 \times (10^{-6} - 10^{-5})$. The first term is roughly an order of magnitude larger than any of the other terms; that is, the percent error is typically less than 10%. Certainly the third term, which is $O(\epsilon^2 q^2)$, is obviously negligible.

Under the right conditions, however, the second term can become important. How might the contribution from the virtual correction [which is dominated by the second term on the rhs of (4)] be significant? There are two dissimilar ways for δq to be at the high end of the estimated range. If the temperature difference δT is large, then, even for a *saturated* environment, δq can be large simply because of the large difference between the parcel and environmental temperatures. In such a case, the parcel would be rising along a moist adiabat with a large associated value of wet-bulb potential temperature (θ_w), whereas the environment would be characterized by a low θ_w. This means that the associated mixing ratio differences between the rising parcel and its environment at any level would be relatively large. For large values of δT, the ratio of $|\epsilon \delta q|$ to $|\delta T q/T|$ can be as large as $(6 \times 10^{-3})/(3 \times 10^{-2}) \sim 20\%$. Such a case is somewhat unlikely; large δT typically occurs with high lapse rates that are almost never observed in nearly saturated conditions (Doswell et al. 1985), but in conditions of convective instability where dry layers with steep lapse rates surmount moist, nearly saturated low-level conditions, the error made can become large.

For small values of δT, δq still can be large when the environment is dry. In such situations, it is conceivable that the virtual correction to the buoyancy could be as large as $(6 \times 10^{-3})/(3 \times 10^{-3}) \sim 200\%$ of the uncorrected value. It should be observed, however, that to have a small δT over a deep layer, the environmental lapse rate cannot be large in comparison to the rising parcel. When the environmental lapse rate is not far from moist adiabatic, it is somewhat unlikely that such an environment also would be extremely dry. Therefore, it is correspondingly unlikely that this extreme sensitivity to the virtual correction would be encountered either. Most often, the error incurred by neglecting the virtual correction would be considerably less than this extreme. An important exception to this rule is when treating *shallow* convection (e.g., ordinary cumulus clouds). For such clouds, the thermal perturbation can be small in the presence of enough δq to make the virtual correction quite important.

There is an additional consideration. The preceding discussion concerns the buoyancy at a particular level. But CAPE is the result of *integrated* buoyancy:

$$\text{CAPE} = g \int_{z_{\text{LFC}}}^{z_{\text{EL}}} \frac{\delta T_v}{T_v}\, dz,$$

where g is the acceleration due to gravity, the subscript "LFC" denotes the *level of free convection,* and the subscript "EL" denotes the *equilibrium level.* The errors made by neglecting the virtual correction do not cancel but accumulate with height because the correction is always of the same sign (positive). However, the magnitude of the correction decreases with height, because the saturation vapor pressure decreases with decreasing temperature. Since virtual corrections typically are on the order of 1°C or less, the *average* correction over a depth of several kilometers is going to be less than 1°C.

The *relative* error E_r due to computing CAPE without the virtual correction is found by

$$E_r \equiv \frac{g \int \frac{\delta T_v}{T_v}\, dz - g \int \frac{\delta T}{T}\, dz}{g \int \frac{\delta T_v}{T_v}\, dz} \approx \frac{g \int (\epsilon \delta q) dz}{\text{CAPE}}, \qquad (5)$$

where the $O(\epsilon^2 q^2)$ term in (4) associated with the virtual correction has been neglected. In (5), it can be seen that, as already observed, when the CAPE is small, the correction can become a significant fraction of the total, whereas for large CAPE, the accumulated error is not likely to be a substantial fraction of the CAPE. The parcel's virtual correction always is larger than the environment's for positively buoyant parcels and might be substantial with large amounts of low-level moisture surmounted by a deep, dry layer. All of the foregoing suggests that the magnitude of the correction should increase with increasing CAPE, in general. As exemplified in Fig. 1a, however, when the buoyancy is large, the *relative* error is small.

If the corrected CAPE happens to be small, the virtual correction should be small but can assume large relative importance (as in the example of Fig. 1b). Suppose the net effect of the virtual correction increases the parcel's buoyancy by an average amount of 0.3°C over the depth of the positive area. This would correspond to a difference in the CAPE of about 100 J kg^{-1} if $z_{\text{EL}} - z_{\text{LFC}} \sim 10$ km. A correction to the CAPE of this amount would not be significant in comparison with the errors associated with the radiosonde thermodynamic measurements if the true CAPE (i.e., including the correction) is about 3000 J kg^{-1}, but it could be important if the true CAPE is on the order of 300 J kg^{-1}.

3. An empirical test

The preceding theory suggests that the relative error is inversely proportional to CAPE, whereas the absolute error should increase with increasing CAPE. To test this concept against the real observations, we evaluated all available soundings within the continental United States for the year 1992. We considered only soundings that have positive values of CAPE when the virtual correction has been made. As expected, the occurrence frequency decreases as CAPE increases (Fig. 2), and the magnitude of the correction shows a general increase as CAPE increases (Fig. 3). The relative error distribution validates the theory presented above (Fig. 4), confirming that large relative errors occur only for small CAPE values. Given the large number of soundings evaluated (5876), significant departures from this basic analysis should be considered rare.

4. Discussion

With the convenience and simplicity of today's computing resources, it is easy to include the effect of the virtual correction when calculating CAPE. Should an existing program to determine CAPE *not* include the virtual correction, a revision of the program may not be an urgent need, depending on the uses for the CAPE calculations. This raises the issue of the purposes to which CAPE estimates from soundings might be applied. If CAPE is used to estimate vertical velocity via pure parcel theory, then there are many problems with such an application: ignoring the virtual correction is only one among the many. Vertical motion estimates also need to consider including the effect of (i) condensed water loading, (ii) entrainment, (iii) nonhydrostatic pressure gradients, and (iv) latent heat associated with the ice phase, if possible. It is not obvious that CAPE is a very useful parameter for anything more than very crude estimates of vertical motion. Those seeking a good estimate of vertical motion would be better off using a one-dimensional cloud model to include the appropriate physical processes than trying to adapt CAPE as a parameter to suit their needs.

The name "convective available potential energy" suggests that an appropriate application would be to estimate the contribution to parcel dynamics from pure parcel theory buoyancy. If this is the agreed-upon application, then we have indicated that the virtual correction should be applied. Even though the error associated with neglecting the virtual correction tends to be small, making the correction is too easy not to do and, in making the correction, the CAPE estimate is done in a way that is physically consistent with the application. Including such esoterica as ice phase contributions and condensate loading seems inappropriate for this general application, although they may well be important contributing factors in particular applications. There does not appear to be any simple way to include them without creating complications well be-

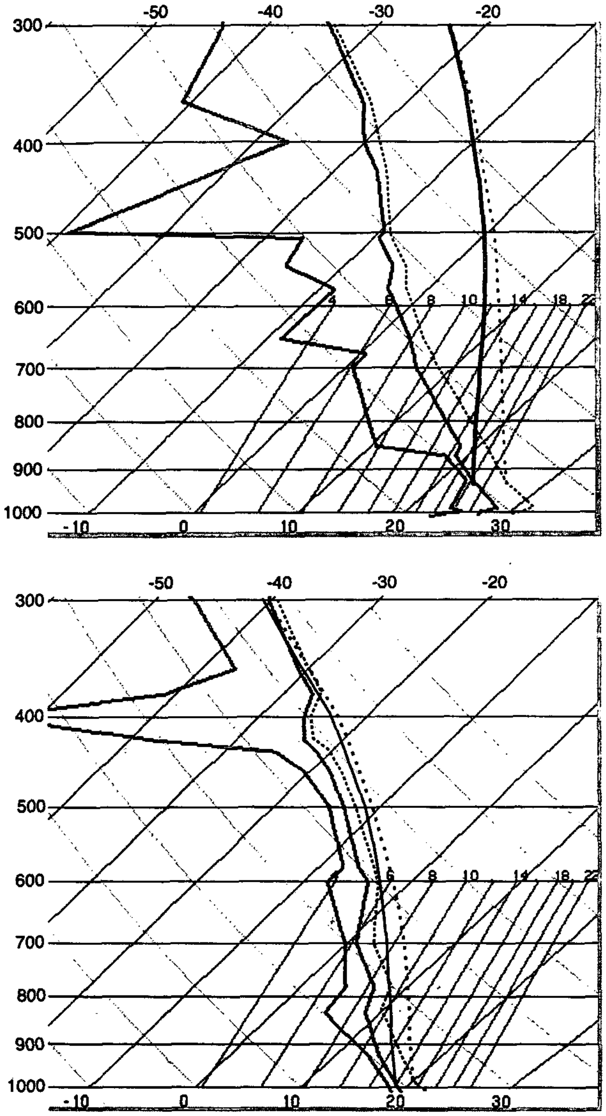

FIG. 1. Examples of soundings plotted on a skew T-log$p$ diagram, illustrating the effect of the virtual correction. The solid lines show the original sounding and the pseudoadiabatic ascent curve for the most unstable parcel in the lowest 300 mb for (a) Wallops Island, Virginia, on 12 August 1992 at 0000 UTC—the corrected CAPE is 4737 J kg^{-1} and the uncorrected value is 4390 J kg^{-1}, giving a difference of 347 J kg^{-1} (7.3% of the corrected value); and (b) Tallahassee, Florida, on 25 February 1992 at 0000 UTC—the corrected CAPE is 402 J kg^{-1} and the uncorrected value is 341 J kg^{-1} for a difference of 61 J kg^{-1} (15.2%). The dashed lines on each show the effect of the virtual correction on the sounding and the rising parcel.

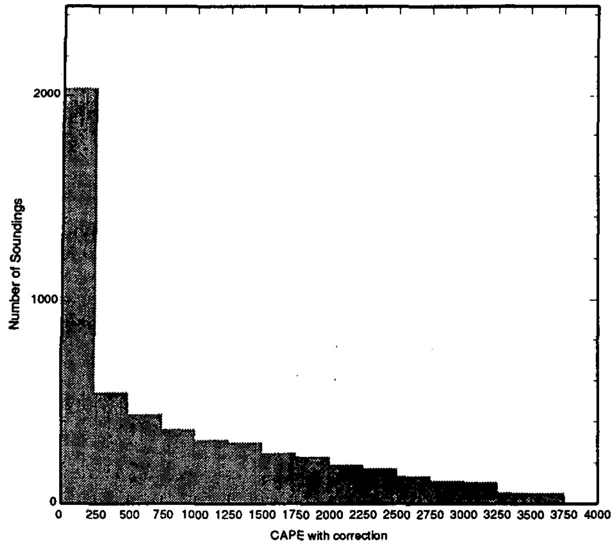

FIG. 2. Histogram showing the occurrence of CAPE values. All soundings showing a positive CAPE (when using the virtual correction) during the year 1992 are plotted; the total number of such soundings is 5876.

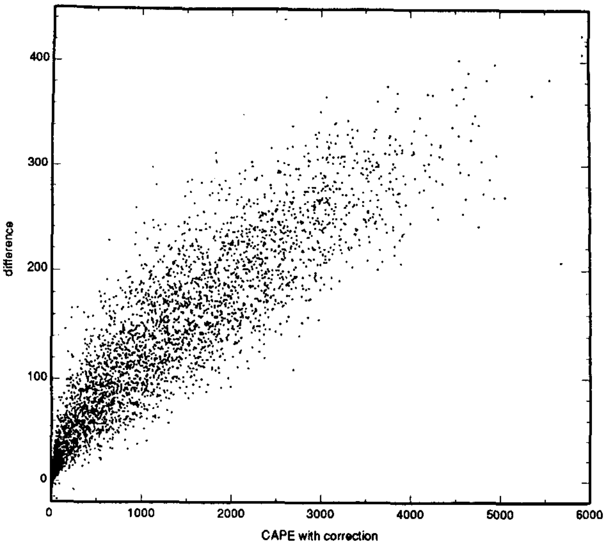

FIG. 3. Scattergram of the differences between CAPE values caused by ignoring the virtual correction as a function of CAPE.

yond the scope of the basic task of estimating available buoyancy.

Users of CAPE in this proposed application should be aware of the imprecision of radiosonde thermodynamic measurements: the virtual correction is typically within the instrumental imprecision. Therefore, the parameter values should not be interpreted too precisely. Certainly one cannot put too much confidence in CAPE differences smaller than 100 J kg^{-1} simply because of measurement inaccuracy.

Moreover, a question of some import, perhaps even greater import than including the virtual correction, is deciding on *which* parcel to lift (see Williams and Renno 1993). This can make a significant difference, even when the CAPE is large (see, e.g., Doswell and Brooks 1993). There does not appear to be much agreement on a standard way to select the parcel to lift. Various schemes choose (a) the surface parcel, or (b) a parcel with the average properties (mixing ratio and potential temperature) within some layer (and the layer depth is not universally agreed upon), or (c) the parcel with the largest CAPE in the lower troposphere. For the lifted index (Galway 1958), a predecessor to CAPE, the lifted parcel has the *forecast* (for the expected time of convection) mean mixing ratio and potential temperature in the lowest 100 mb of the sounding. This artifice obviously is designed to account for some of the diurnal variation.

We propose the following standard for using CAPE as a forecasting tool and to make comparisons from one sounding to another

(i) Use the simple pseudoadiabatic, pure parcel theory calculation for the lifted parcel ascent.

(ii) Make the virtual correction.

(iii) Pick *the most unstable parcel* in the lowest 300 mb.

The use of parcels having properties of some well-mixed sublayer always involves arbitrary choices about what layer depth to use, and it is hard to get consistency among users on the chosen layer depth. Choosing the most unstable parcel has the advantage of being applicable when surface-based parcels or layers are clearly inappropriate, as in nocturnal convection. We rec-

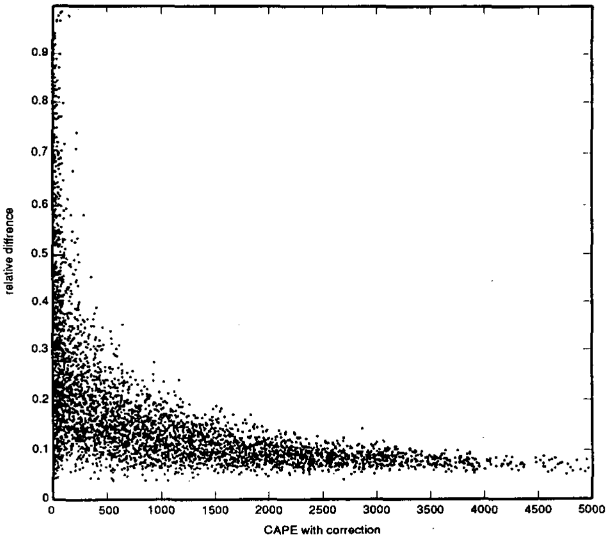

FIG. 4. Plot of the relative difference in the CAPE with and without the virtual correction for the 1992 positive CAPE soundings (as in Fig. 2) versus CAPE (with correction).

ommend ignoring all other complexities unless a particular application requires them directly. If someone wants to evaluate a sounding constructed either by *extrapolation* as in a forecast or by *interpolation* in between stations, then we recommend using the proposed standard method on the constructed sounding. This is in contrast to what is done with the lifted index, which frequently is interpreted to represent the instability associated with a given sounding but really represents the instability of a *forecast* sounding.

A final issue concerns the computation of convective inhibition (CIN) associated with a rising parcel's low-level, negatively buoyant area typically found in observed soundings. CIN values tend to be relatively small in convective situations, and they occur low in the sounding, where the virtual corrections can be approximately 1 K. Of course, integration layer depths are correspondingly lower, which contributes to small CIN values. If the integration depth is only 1 km but the virtual correction's *change* in the buoyancy averages 1 K, the corresponding effect on CIN would be approximately 35 J kg^{-1}. For a CIN of -100 J kg^{-1}, this is an important contribution. Since programs to compute CIN typically are the same programs used to compute CAPE, this is an argument in favor of changing to include the virtual correction if the algorithm in use does not do so.

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